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G. Barenboim, P. Ko & **WIP**, arXiv: 2403.05390,
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The MCSM
(The Minimal Cosmological Standard Model)

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Motivation

- **Big theoretical/cosmological/phenomenological puzzles**

- The origin of dimensionful parameters ($m_{\rm P} = 1/\sqrt{G}$ & $\mu_h = m_h/\sqrt{2}$)

- Inflation ($n_s \simeq 1 - 0.03,\ r_T < 0.063$)
- Matter-antimatter asymmetry ($Y_B \approx 10^{-10}$)
- Dark matter ($\Omega_{\rm DM} \sim 0.27$)
- H_0 tension ($\left(H_0^{\rm SN} - H_0^{\rm CMB}\right)/H_0^{\rm CMB} \sim 0.1$ ($> 4\sigma$ dev.))
- Cosmological constant ($\rho_\Lambda^{\rm obs}/\rho_\Lambda^{\rm th} \lesssim \rho_\Lambda^{\rm obs}/\Lambda_{\rm QCD}^4 \sim 10^{-43}$)

- Hierarchy problem ($\left(m_{\rm ew}/M_{\rm P}\right)^2 = \mathcal{O}(10^{-34})$)
- Neutrino mass ($m_\nu/m_{\rm ew} = \mathcal{O}(10^{-13})$)
- Strong CP-problem ($\theta_{\rm CP} < \mathcal{O}(10^{-10})$)

- … (?)

mostly being separately tackled!

- **A dream (or a goal)**

- A simple unified BSM framework for all(or most) of them?

(The philosophy: nature works in the most efficient way if possible!)

- **Simple guiding principles for the goal (?)**

(i) Scale symmetry ⇒ Dynamical generation of scales w/o add-hoc mass parameters.
(ii) $U(1)_{\rm PQ}$ symmetry (at least in the matter sector) ⇒ Axion-solution
(iii) Minimality ⇒ Unification of PQ & seesaw sectors (⇒ "Axi-Majoron")

< Minimal DFSZ vs minimal KSVZ >

✓ **DFSZ**:
One extra Higgs-doublet (w/ the danger of the domain-wall problem)

KSVZ:
A pair of extra quark-triplets (w/o the domain-wall problem)

Global Weyl scaling [Weyl, 1921]

Transformation:

$$g_{\mu\nu} \equiv \Omega^{-1}\tilde{g}_{\mu\nu}\ , \quad \left(\Omega \equiv e^{-2\sigma/f}\right)$$

$$\begin{cases} \phi \equiv \Omega^{1/2}\tilde{\phi} \\ \psi \equiv \Omega^{3/4}\tilde{\psi} \\ e^{\mu}_{\alpha} \equiv \Omega^{1/2}\tilde{e}^{\mu}_{\alpha} \;\; (\gamma^{\mu} = \gamma^{\alpha} e^{\mu}_{\alpha}) \text{: vielbein} \\ R = \Omega\left[\tilde{R} - \frac{3}{2}\left(\partial \ln \Omega\right)^2 + 3\tilde{\nabla}^2 \ln \Omega\right] \text{: Ricci scalar} \end{cases}$$

Scale invariant action ($d = 4$ operators w/o dimensionful parameters):

$$S = S_G + S_M\ ,$$

$$S_G \equiv -\frac{1}{2}\int d^4x\sqrt{-g}R\mathcal{F}(\{\varphi_i^2\})\ ;\ \mathcal{F} \equiv \sum_i \xi_i\varphi_i^2 \;\; (\xi_i > 0)$$

$$S_M = \text{Matter-action without mass-parameters}$$

Nonminimal gravitational int.

Noether current & its conservation:

$$K_\mu \equiv \sum_i \kappa_i\varphi_i\partial_\mu\varphi_i \;\; (\kappa_i \equiv 1 + 6\xi_i)$$

$$D_\mu K^\mu = 4V - \sum_i \varphi_i\frac{\partial V}{\partial\varphi_i} = 0$$

→ global Weyl scale-invariant!

Shaposhnikov & Zenhousern, PLB671 (2009) 162
Ferreira, Hill & Ross, PRD95 (2017) 043507

Quantum scale-symmetry

- Trace anomaly: Due to a fixed input scale μ_0 for RG-running.

$$V = \frac{\lambda}{4}\varphi^4 + \frac{\beta_\lambda}{4}\varphi^4 \ln\left(\frac{\varphi}{\mu_0}\right)$$

$$\Rightarrow \quad D_\mu K^\mu = 4V - \varphi\frac{\partial V}{\partial \varphi} = -\frac{\beta_\lambda}{4}\varphi^4 \neq 0$$

$\Rightarrow$ Scale-invariant only if $\beta_\lambda = 0$

- Restoration of scale-sym.: Replacing μ_0 to a Weyl co-variant field such as χ.

$$V = \frac{\lambda}{4}\varphi^4 + \frac{\beta_\lambda}{4}\varphi^4 \ln\left(\frac{\varphi}{\chi}\right)$$

$$\Rightarrow \quad D_\mu K^\mu = 4V - \varphi\frac{\partial V}{\partial \varphi} - \chi\frac{\partial V}{\partial \chi} = 0$$

$\Rightarrow$ Scale-invariant even though $\beta_\lambda \neq 0$

Scale generation

The Weyl-current, a kernel & its asymptote [Ferreira, Hill & Ross, PRD95, 043507 (2017)]

$$K_\mu = \sum_i \kappa_i \varphi_i \partial_\mu \varphi_i = \partial_\mu K \quad \left(K \equiv \sum_i \kappa_i \varphi_i^2/2, \;\; \kappa_i \equiv 1 + 6\xi_i \right)$$

$$D^\mu K_\mu = 0 \;\Rightarrow\; \ddot{K} + 3H\dot{K} = 0$$

$$\Rightarrow\; K = c_1 + c_2 \int \frac{dt}{a^3(t)} \xrightarrow{t\to\infty} c_1 \;\; \text{(a const.!)}$$

A scale appears!

⇨ Dynamic scale-generation!
Potential causes a field-dynamics, but with $K = $ const.

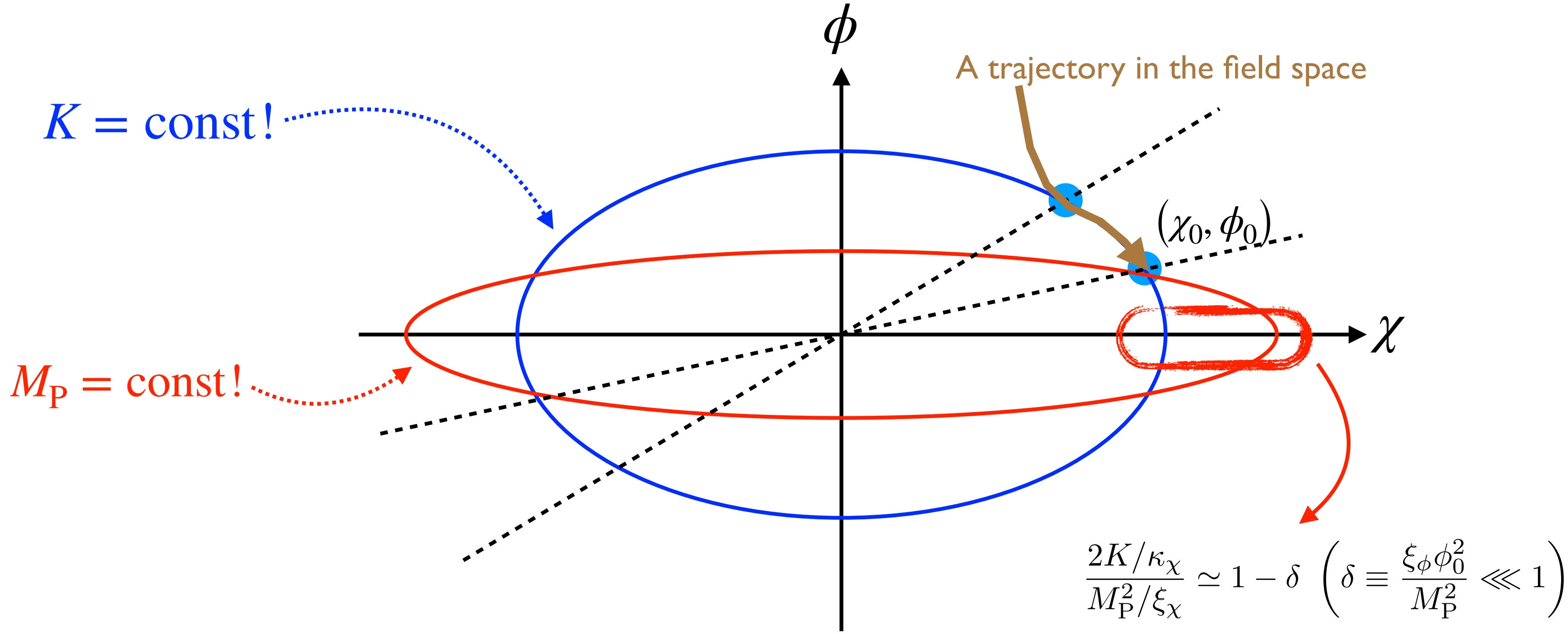
A simple two-field picture (with $\xi_\chi \lll 1$): [Ferreira, Hill & Ross, PRD95, 043507 (2017)]

$$K = \kappa_\chi \chi^2 + \kappa_\phi \phi^2 \simeq \chi^2 \left(1 + \frac{\kappa_\phi \phi^2}{\chi^2}\right) \to \text{const}\,.$$

$$M_{\rm P}^2 = \xi_\chi \chi^2 \left(1 + \frac{\xi_\phi \phi^2}{\xi_\chi \chi^2}\right) \xrightarrow{\xi_\phi^{1/2}\phi \lll \xi_\chi^{1/2}\chi} \xi_\chi \chi^2 \simeq M_{\rm P,0}^2$$

$$\sqrt{\kappa_\phi} \ll \frac{\chi_{\rm ini}}{\phi_{\rm ini}} \ll \sqrt{\xi_\phi/\xi_\chi} \;\Rightarrow\; \begin{cases} K \simeq \chi_{\rm ini}^2 & : \text{ always} \\ M_{\rm P}^2 \simeq \xi_\phi \phi_{\rm ini}^2 \gg M_{\rm P,0}^2 & : \sim \text{the SM Higgs inflation} \end{cases}$$

$\Rightarrow$ A large-field inflation can be realized in the same manner as the SM Higgs inflation.



- **Decoupling of the dilaton** [Ferreira, Hill & Ross, PRD95, 064038 (2017)]

For scalar fields as an example,

$$\begin{cases} g_{\mu\nu} = e^{2\sigma/f}\tilde{g}_{\mu\nu} \quad (\sigma = \text{dilaton}) \\ \varphi_i = e^{-\sigma/f}\tilde{\varphi}_i \\ f^2 \equiv \overline{K} = \sum_i \kappa_i \tilde{\varphi}_i^2/2 = \text{const!} \;\; (f \simeq \chi_0 \simeq M_{\rm P}/\xi_\chi \gg M_{\rm P}) \end{cases}$$

$$\begin{cases} S_{\rm G} = -\frac{1}{2}\int \sqrt{-\tilde{g}}\tilde{\mathcal{F}}\left[\tilde{R} - \frac{6}{f^2}(\partial\sigma)^2 - \frac{6}{f}\tilde{\nabla}^2\sigma\right] \;\; \left(\tilde{\mathcal{F}}(\{\tilde{\varphi}_{i,0}\}) = M_{\rm P}^2\right) \\ S_{\rm M}^{\rm s} = \int\sqrt{-\tilde{g}}\left\{\frac{1}{2}\sum_i\left[(\partial\tilde{\varphi}_i)^2 + \frac{\tilde{\varphi}_i^2}{f^2}(\partial\sigma)^2 - \frac{1}{f}\partial\sigma\cdot\partial\left(\tilde{\varphi}_i^2\right)\right] - \tilde{V}(\{\tilde{\varphi}_i\})\right\} \end{cases}$$

$$\Rightarrow \;\; S_{\rm tot} = \int\sqrt{-\tilde{g}}\left\{\cdots + \frac{\overline{K}}{f^2}(\partial\sigma)^2 + \frac{1}{f}\partial\sigma\cdot\partial\overline{K} + \cdots + \lambda_L\mathcal{C}(\{\tilde{\varphi}_i\})\right\} \quad (\mathcal{C} = \overline{K} - f^2)$$

- σ is massless!
- No derivative interactions ($\because$ scale-invariance $\Rightarrow \overline{K} = f^2 = \text{const}.$).
- It is completely decoupled (cancelled) from the matter sector potential.

$\Rightarrow$ No fifth force constraints!

A full scale-invariant theory can be consistent with low energy phenomenology!

The MCSM

(The Minimal Cosmological Standard Model)

[Barenboim, Ko & **Park**, 2403.05390; PRD11(2024)12,123521]

• Underlying symmetries

- Scale-invariance
- $U(1)_{\rm PQ}$ (but broken in the gravity sector)

• Minimal field contents & charges

$$\left\{\begin{aligned} \chi &= \text{A real scalar}: \ \langle\chi\rangle \to M_{\rm P} \\ \Phi &= \text{the Peccei-Quinn field(a complex scalar)}: \ \langle\Phi\rangle \to U(1)_{\rm PQ}\text{-breaking} \\ H_2 &= \text{an additional Higgs doublet}: \ \text{DFSZ-axion model} \\ \nu_{R_i} &= \text{three right-handed neutrino fields}: \ \text{Seesaw mechanism} + \text{reheating} \end{aligned}\right.$$

Field / Charge	Q_L	u_R	d_R	ℓ_L	e_R	ν_R	H_1	H_2	Φ
$q_{\rm PQ}$	3/2	1/2	1/2	3/2	1/2	1/2	1	-1	1

• The model (Axi-majoron + non-minimal grav.-coupling)

Key terms of our scenario!

$$\mathcal{L} = \mathcal{L}_{\rm g} + \mathcal{L}_{\rm m}$$

$$\mathcal{L}_{\rm g} = -\frac{1}{2}\tilde{R}\left[\xi_\chi \chi^2 + \xi_\Phi |\Phi|^2 + \xi_+ \left(\Phi^2 + {\rm c.c.}\right) - i\xi_- \left(\Phi^2 - {\rm c.c.}\right) + \mathcal{F}_H(H_1, H_2)\right] + {\rm G.B.}$$

$$\mathcal{F}_H = \xi_{H_1}|H_1|^2 + \xi_{H_2}|H_2|^2 + \xi_+^H\left(H_1^\dagger H_2 + {\rm c.c.}\right) - i\xi_-^H\left(H_1^\dagger H_2 - {\rm c.c.}\right)$$

$$\mathcal{L}_{\rm m} \supset \mathcal{L}_{\rm ss} - \tilde{V}_S$$

Simplified version with only one Higgs doublet:

$$\tilde{V}_S = \frac{\lambda_\chi}{4}\chi^4 + \lambda_h|H|^4 + \lambda_\phi|\Phi|^4 - \frac{1}{2}\lambda_{\chi h}\chi^2|H|^2 - \frac{1}{2}\lambda_{\chi\phi}\chi^2|\Phi|^2 - \lambda_{h\phi}|H|^2|\Phi|^2$$

$$\mathscr{L}_{\rm ss} = \frac{1}{2}y_N \Phi^* \overline{\nu_R^c}\nu_R + y_\nu \bar{\ell}_L \tilde{H}\nu_R + {\rm h.c.}\ .$$

(⇒ "Axi-majoron" model)

- Dynamically relevant three fields with $\langle H\rangle \sim 0$:

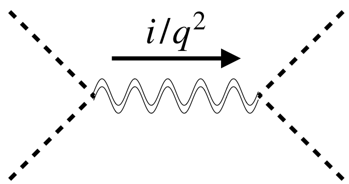
$$\varphi_\alpha = (\chi,\ \phi_r,\ \phi_i)\ \left({\rm for}\ \Phi \equiv (\phi_r + i\phi_i)/\sqrt{2} = \phi e^{ia_\phi/\langle\phi\rangle}/\sqrt{2}\right)$$

• Axion quality problem(?)

Grav. non-pert. effect: [Kallosh, Linde, Linde & Suskind, PRD 52 (1995) 912]
Gauss-Bonnet term may retain the corrections negligible.

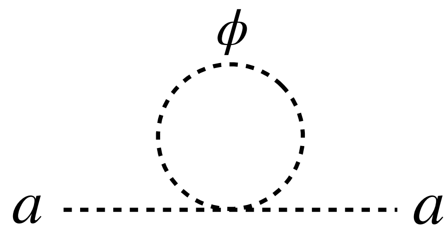
Perturbative effect: [Hill and Ross, PRD102, 125014(2020)]
Impact of the dangerous graviton exchanges caused by the sym.-breaking terms:

$$S=\int \sqrt{-g}\left(\frac{1}{2}M_P^2 R(g_{\mu\nu})+\frac{1}{2}F(\phi_i)R(g_{\mu\nu})+L(\phi_i)\right)$$



$$\frac{(q^2)^2}{q^2} \to q^2$$

$$S_{CT} = \int d^4x \frac{3\xi^2}{M_P^2}\phi^2 \partial\phi\partial\phi + \int d^4x \frac{\xi\phi^2}{2M_P^2}\left(-\partial^\sigma\phi\partial_\sigma\phi + 4W(\phi)\right)$$



$$\frac{\Delta m_a^2}{m_{a,\mathrm{QCD}}^2} \sim \alpha\xi_\phi^2\left(\frac{m_\phi^4}{m_{a,\mathrm{QCD}}^2 M_\mathrm{P}^2}\right)\ln\left(\frac{\chi^2}{m_\phi^2}\right) \gg 1$$

$$m_\phi \sim \sqrt{\lambda_\phi}\phi_0 \gtrsim \mathcal{O}(10^7)\mathrm{GeV}$$

$$\alpha = \xi_{\mathrm{sym-breaking}}/\xi_{\mathrm{sym}}$$

A cure

$$\xi_a \to \xi_a = \xi_{a,0}e^{-c_a\chi/|\varphi_a|} \qquad \begin{cases} c_a\chi \ll |\varphi| & : \text{ during inflation} \\ c_a\chi \gg |\varphi_0| & : \text{ after inflation} \end{cases}$$

- **The origin of scales**

χ is nearly fixed, once the kernel is fixed

$$\tilde{V}_S(\chi,h,\phi)=\frac{\lambda'_\chi}{4}\chi^4+\frac{\lambda'_\phi}{4}\left(\phi^2-\zeta_{\phi\chi}\chi^2\right)^2+\frac{\lambda_h}{4}\left(h^2+\zeta_{h\phi}\phi^2-\zeta_{h\chi}\chi^2\right)^2$$

$$\Downarrow\quad (\xi_\chi\chi^2\to M_{\rm P}^2)$$

$$V(\chi,\phi,h)\equiv\frac{\tilde{V}_S}{\Omega^2}\approx V_0+\frac{\lambda'_\phi}{4}\left(\phi^2-\phi_{0,\chi}^2\right)^2+\frac{\lambda_h}{4}\left(h^2+\zeta_{h\phi}\phi^2-h_{0,\chi}^2\right)^2$$

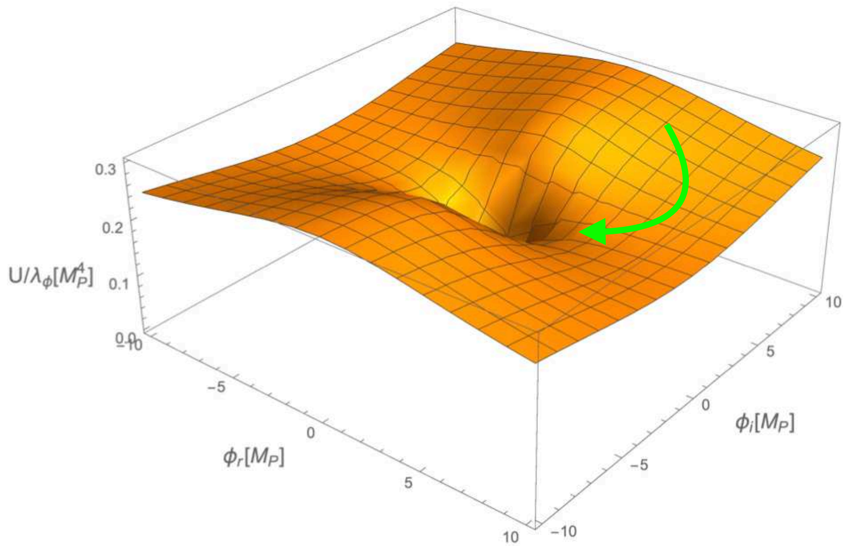
$$V_0=\frac{\lambda'_\chi}{4\xi_\chi^2}M_{\rm P}^4+\text{1-loop corrections}$$

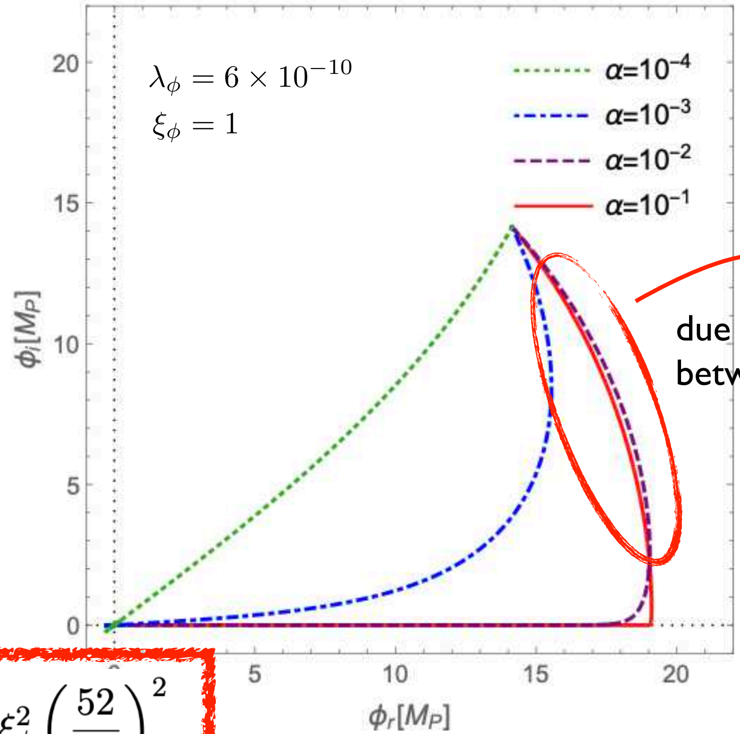
1. χ is responsible for all of mass scales (except the dimensional transmutation).
2. C.C. appears after the spontaneous breaking of the scale-sym. at least due to λ'_χ-term.
3. C.C. problem is now of the choice of λ'_χ/ξ_χ^2.
   (see however PLB671 (2009) 162; 187 (unimodular gravity) & T. Kugo's talk at SI2009)

Cosmology

- **Axi-Majoron Hybrid Inflation**

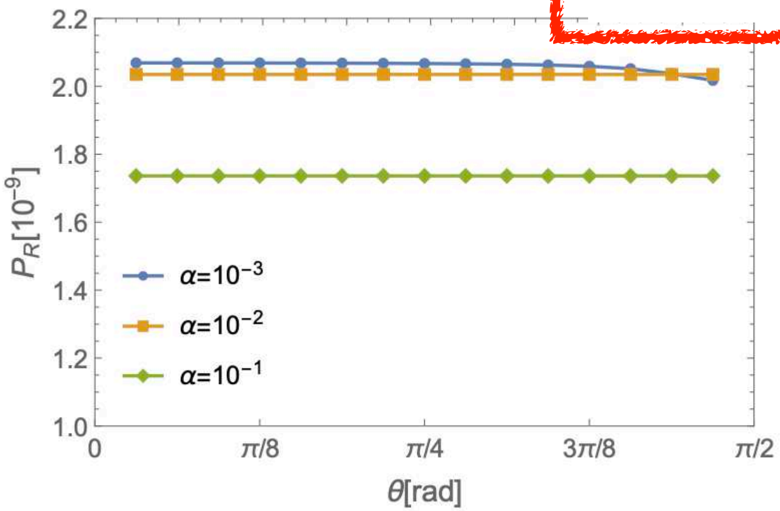
(however effectively single-field inflation)

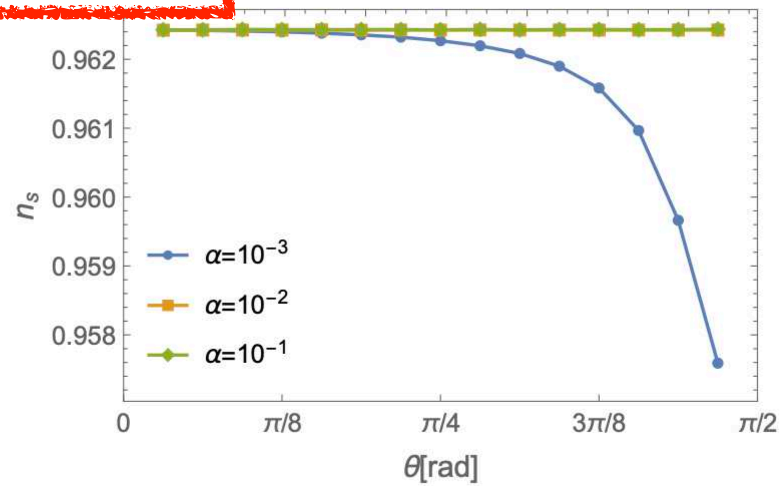




due to the mixing between ϕ_r and ϕ_i

$$\lambda_\phi \sim 6 \times 10^{-10} \xi_\phi^2 \left(\frac{52}{N_e}\right)^2$$

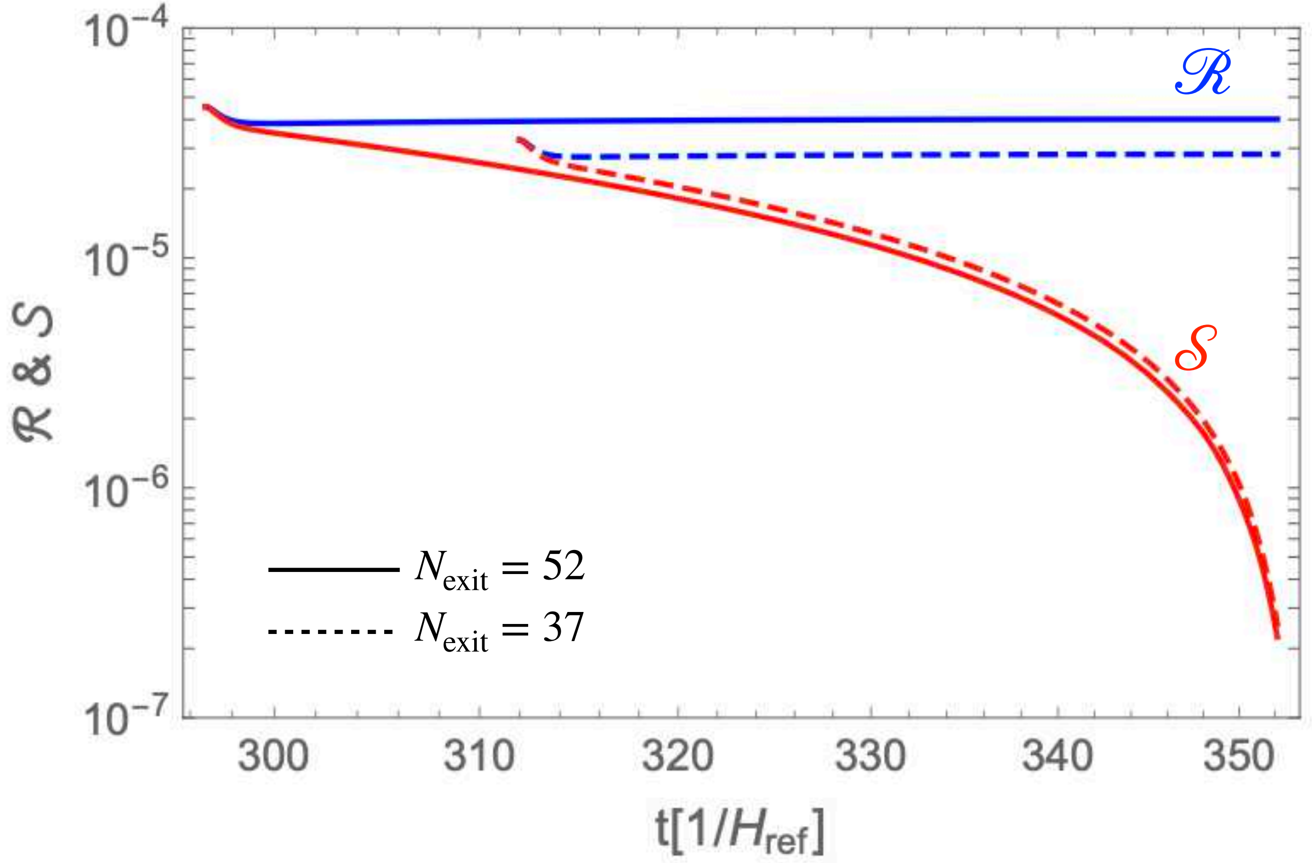




Iso-curvature perturbations:

- Evolution of ϕ causes a suppression ($\dot{\phi}$ increases toward the end of inflation):

$$S \equiv \frac{H}{\dot{\phi}} (\delta\phi^a)_\perp \ \& \ (\delta\phi^a)_\perp \sim \text{const.} \Rightarrow \frac{P_I}{P_R} \propto \frac{\epsilon_{\text{exit}}(k)}{\epsilon_{\text{end}}} \sim \epsilon_{\text{exit}}(k) \sim \mathcal{O}(N_{\text{exit}}^{-2}(k))$$



- **Reheating after inflation**

Relevant interactions:

$$\mathcal{L} \supset \left(\theta - \frac{c_a a_\phi}{\phi_0}\right) \frac{g_s^2}{32\pi^2} G\tilde{G} - V$$

$$V \supset \frac{\lambda_\phi}{4}\left(\phi^2 - \phi_0^2\right)^2 + \left(\frac{1}{2} y_N \Phi^* \overline{\nu_R^c} \nu_R + y_\nu \bar{\ell}_L \tilde{H} \nu_R + \text{h.c.}\right)$$

$$m_\phi = \begin{cases} \sqrt{3\lambda_\phi}\phi & : \ \phi \gg \phi_0 \\ \sqrt{2\lambda_\phi}\phi_0 & : \ |\delta\phi \equiv \phi - \phi_0| \ll \phi_0 \end{cases}$$

(i) ϕ decays dominantly to a_ϕs(axi-majorons) via the kinetic term.

(ii) There would be effects of preheating by axi-majoron-dynamics (under investigation!)

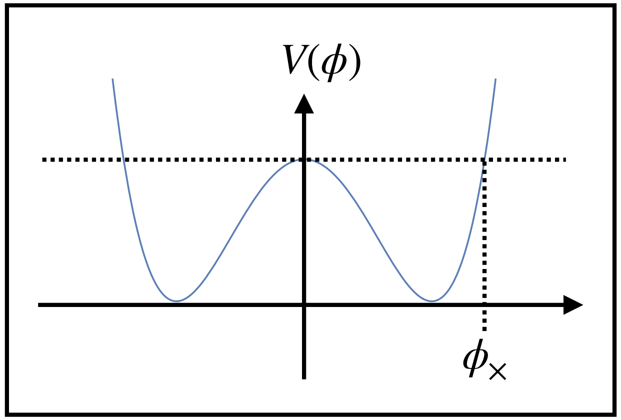
(iii) It should be able to decay to RHNs ($\Rightarrow y_{N_i} < \sqrt{\lambda_\phi}$), too.

(iv) Recovering the SM thermal bath requires a period of MD era due to long-lived RHNs.

$$\Rightarrow \quad \frac{m_{\nu_1}}{m_\nu} < 2 \times 10^{-4} \times \left(\frac{\Delta N_{\rm eff}^{\rm obs}}{0.5}\right)^{3/2} \left(\frac{\xi_\phi}{10}\right) \left(\frac{B_1}{0.1}\right)^2 \left(\frac{\phi_0}{\phi_0^{\rm ref}}\right)$$

$$\left(m_\nu \equiv 0.05 \text{eV}, \ \phi_{\rm ref} \equiv 10^{12} \text{GeV}\right)$$

- **Symmetry non-restoration of $U(1)_{\rm PQ}$**



Preheating:

[Greene et al, PRD56 (1997) 6175; Greene & Kofman, PLB448 (1999) 6]

(i) To ϕ-quanta (due to λ_ϕ-term): $\Delta\rho_\phi = \mathcal{O}(10^{-3})\rho_\phi$

(ii) To ν_Rs: $\Delta\rho_N = \mathcal{O}(0.1) \times b_N^{5/4}\lambda_\phi\rho_\phi \lll \rho_\phi$

$\Rightarrow$ Energy transfer to ν_R via preheating is inefficient!

Thermal trapping?:

(i) Scatterings of ϕ to ν_Rs are inefficient.

$$\left.\begin{array}{l}\Gamma_s \sim y_N^4 T \lesssim \lambda_\phi^2 T \\ H(T) \gtrsim T^2/M_{\rm P}\end{array}\right\} \Rightarrow \Gamma_s/H(T) \lesssim \lambda_\phi^2 M_{\rm P}/T \ \ (\text{at least for } T \gtrsim \phi_0)$$

(ii) The effective mass-squared at the origin when $\phi \sim \phi_* \gtrsim \phi_\times$:

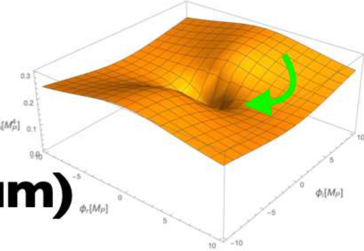
$$m_{\phi,\rm eff}^2(0) \approx \lambda_\phi\phi_0^2\left[-1 + c_T\lambda_\phi^{5/2}\left(\frac{9}{32\pi}\frac{M_{\rm P}}{\phi_0}\right)^2\sum_i b_i\right] < 0 \quad \Rightarrow \quad \xi_\phi \lesssim 680\left(\frac{0.1}{b_N}\right)^{1/5}\left(\frac{\phi_0}{\phi_0^{\rm ref}}\right)^{2/5}$$

$\therefore$ It is possible to have the symmetry not restored.

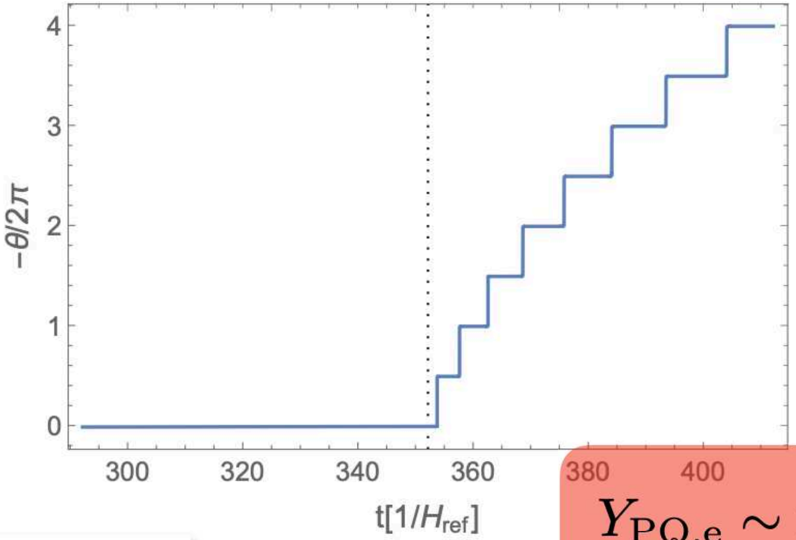
$\Rightarrow$ We can avoid the domain-wall problem of the minimal DFSZ axion model!

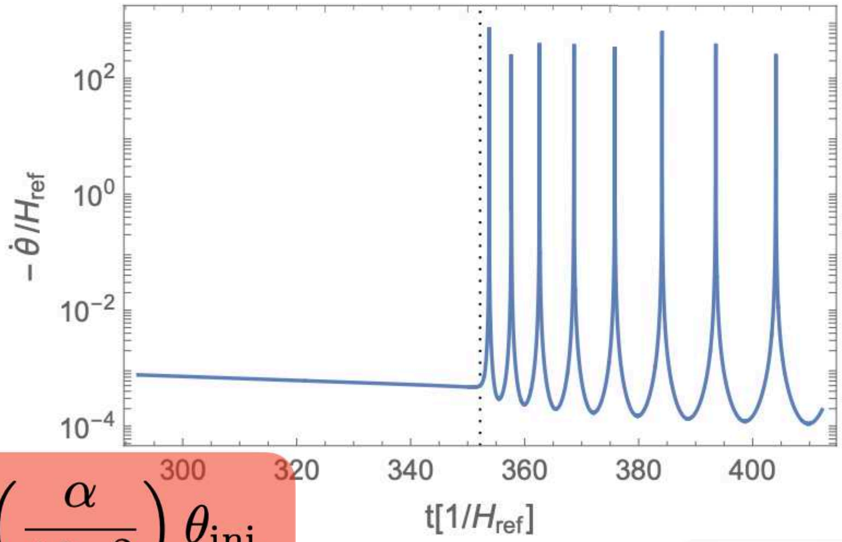
(Potential danger - sym,-breaking potential might cause very efficient preheating)

- **Matter-antimatter asymmetry**

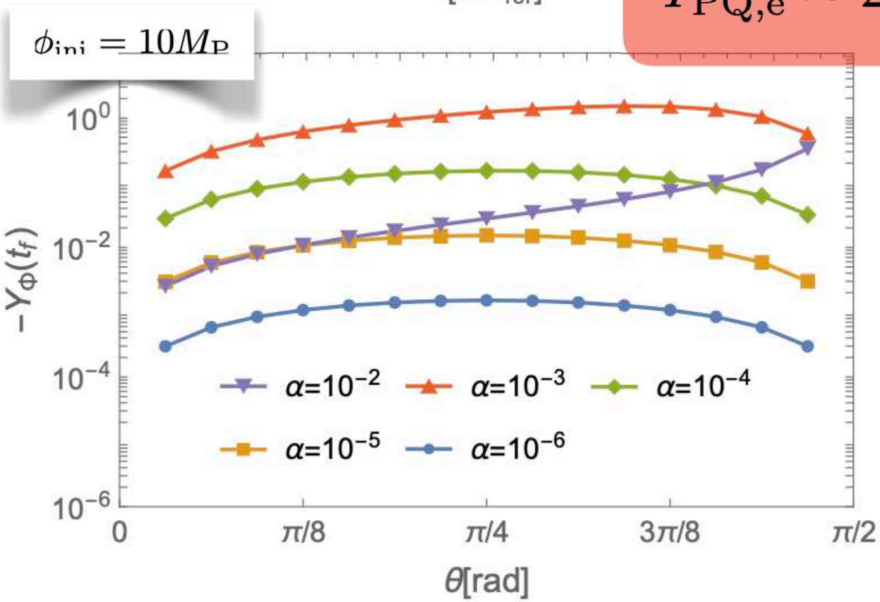


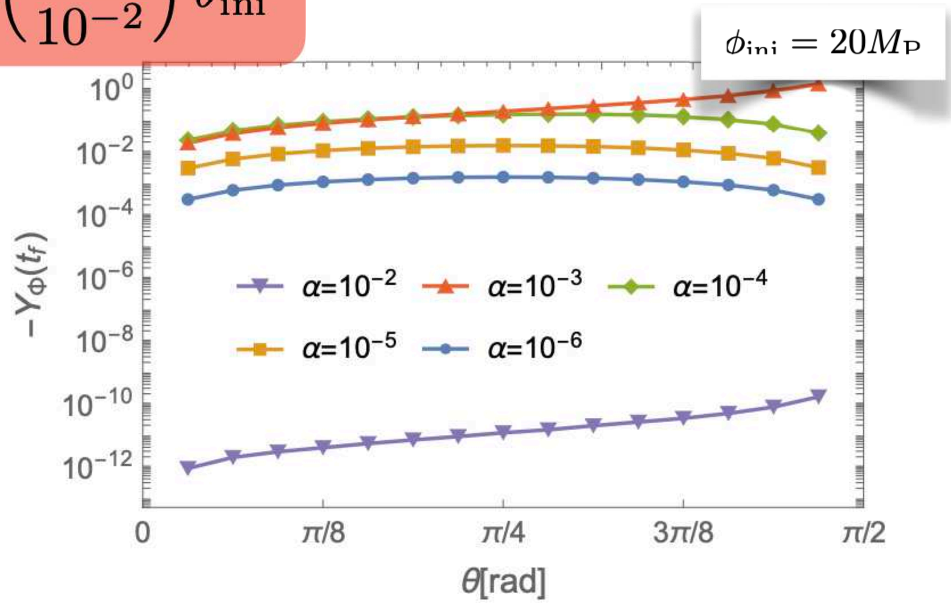
Affleck-Dine PQ-# generation (= non-zero angular momentum)



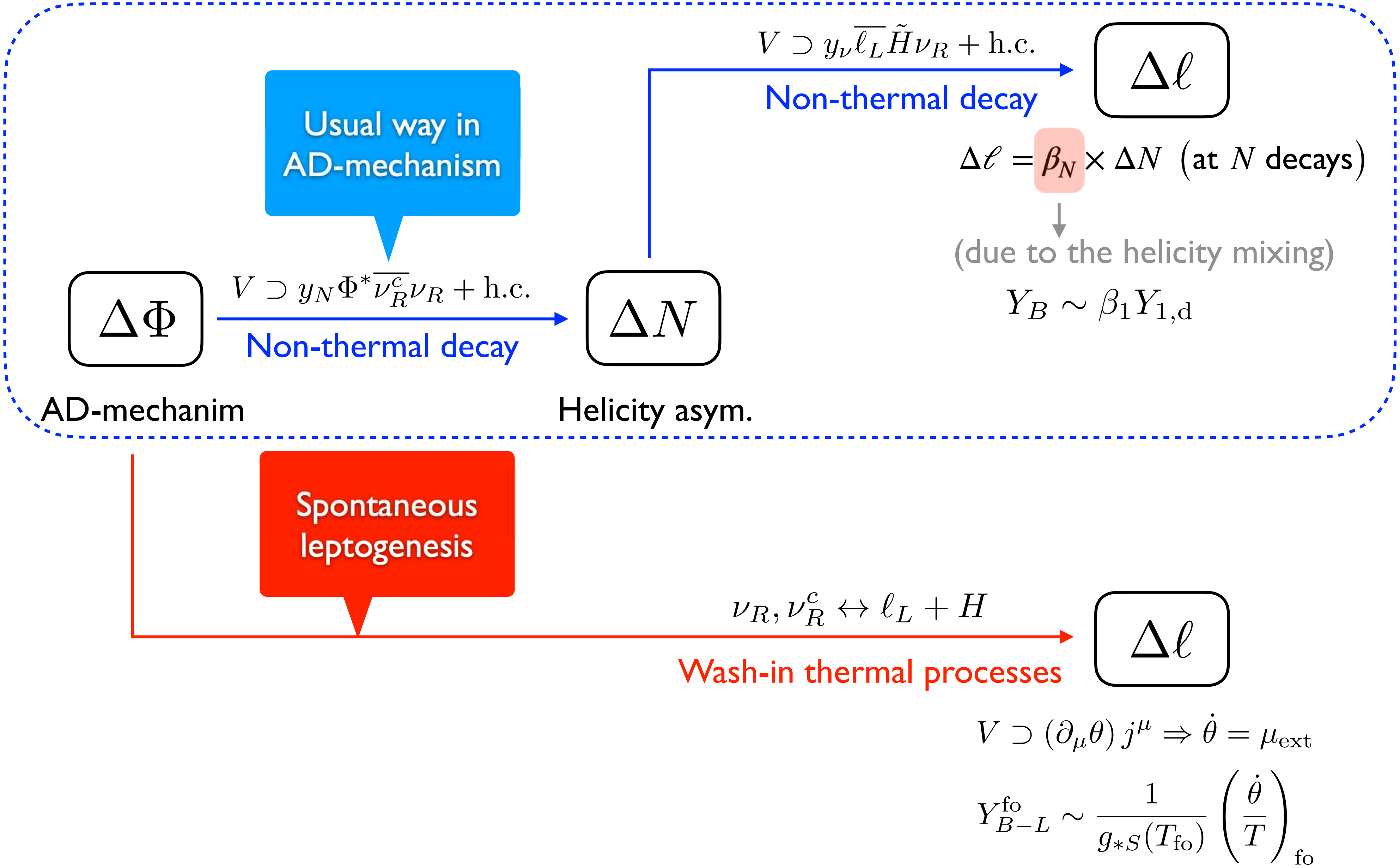


$$Y_{\rm PQ,e} \sim 2.5 \left(\frac{\alpha}{10^{-2}}\right) \theta_{\rm ini}$$





Transferring PQ-number to the SM sector



Helicity asymmetry [Beno & Santos, PRD71 (2005) 096001]

Helicity states are sol. to the Dirac eq. of free RHNs.

One may observe that in contrast to lepton number, helicity is not invariant under Lorentz transformations. However, in an isotropic Universe the comoving thermal bath frame is a privileged frame where isotropy enforces the spin density matrix to be diagonal in the helicity basis. That means that each of the neutrino flavors N_a can be divided in two populations of opposite helicities and well defined distribution functions $f_a^{\pm}$. The total lepton number carried by each neutrino species is equal to

$$L_a = V \int \frac{d^3 p_a}{(2\pi)^3} (f_a^+ - f_a^-) v_a, \qquad (9)$$

where v_a is the neutrino speed and V the spatial volume. It

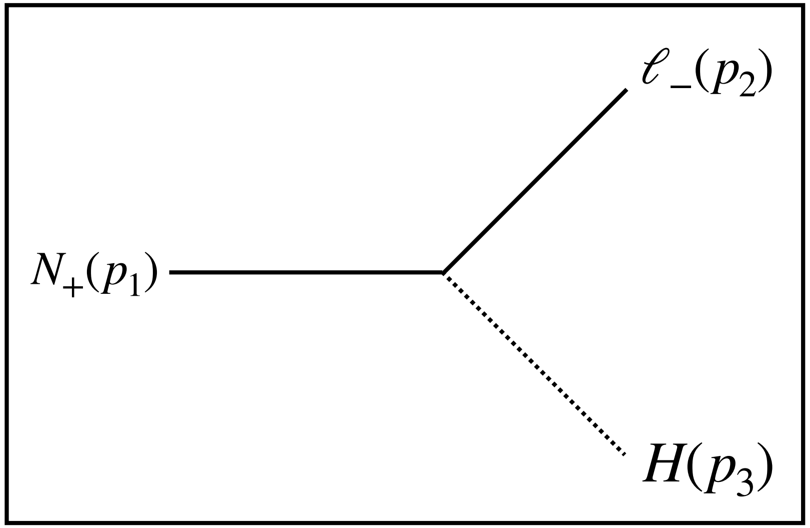
Projectors on components with definite energy and helicity:

$$\Lambda_+^h(p) = \left(\frac{m + \not{p}}{2m}\right)\left(\frac{1+\gamma^5 \not{s}_h}{2}\right) = \frac{u^{(h)}(p)\,\overline{u^{(h)}}(p)}{2m},$$

$$\Lambda_-^h(p) = \left(\frac{m - \not{p}}{2m}\right)\left(\frac{1+\gamma^5 \not{s}_h}{2}\right) = -\frac{v^{(h)}(p)\,\overline{v^{(h)}}(p)}{2m},$$

with the polarization four-vector

$$s_h^\mu = h\left(\frac{|\vec{p}|}{m}, \frac{E}{m}\frac{\vec{p}}{|\vec{p}|}\right), \qquad s_h^2 = -1, \qquad s_h \cdot p = 0,$$



$$\Gamma_{+-} \equiv \Gamma(N^+ \to H + \ell^-) \propto E_1 E_2 (1+\beta_1)$$

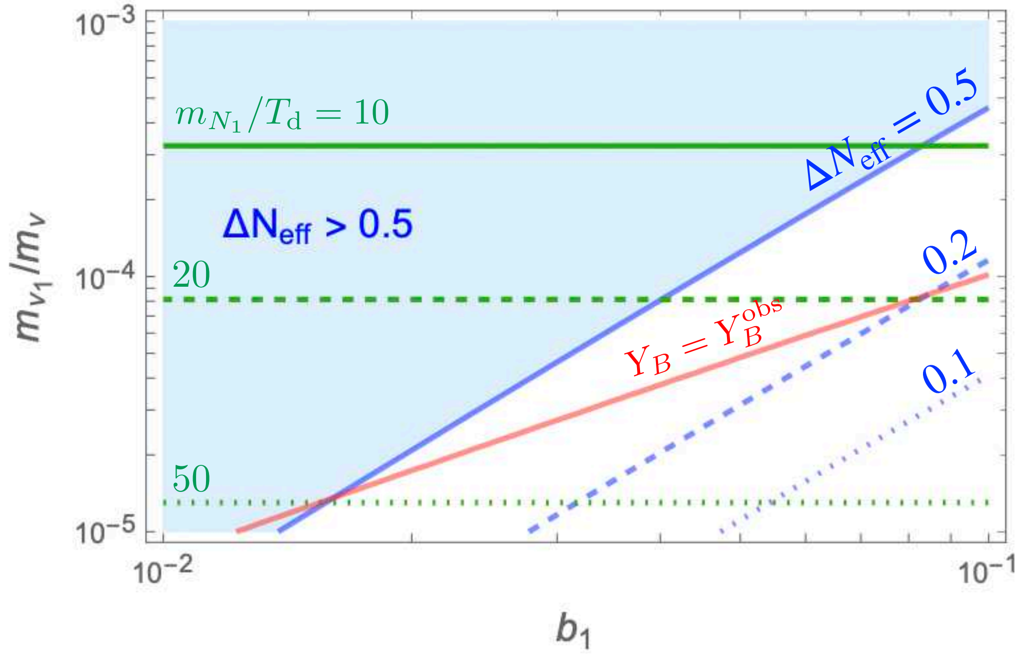
$$\Gamma_{++} \equiv \Gamma(N^+ \to \tilde{H} + \ell^+) \propto E_1 E_2 (1-\beta_1)$$

$$\left.\begin{array}{l} B_{+-} = \frac{1}{2}(1+\beta_1) \\ B_{++} = \frac{1}{2}(1-\beta_1) \end{array}\right\} \Rightarrow \begin{array}{l} \Delta n_\ell = (B_{+-} - B_{++})\,\Delta n_N \\ \quad = \beta_1 \Delta n_N \end{array}$$

BAU through cascade decays: $\Delta\Phi \to \Delta N \to \Delta\ell$

$$Y^{>}_{B,\mathrm{AD}} = \frac{12}{37} \times \beta^{>}_{1,\mathrm{d}} Y^{>}_{1,\mathrm{d}}$$
$$\simeq 6.2 \times 10^{-11} \times \left(\frac{0.1}{\mathrm{B}_1}\right)^{4/3} \left(\frac{\xi_\phi}{200}\right)^{7/6} \left(\frac{10^4 m_{\nu_1}}{m_\nu}\right)^{7/6} \left(\frac{Y_{\Phi,e}}{10^{-6}}\right)$$

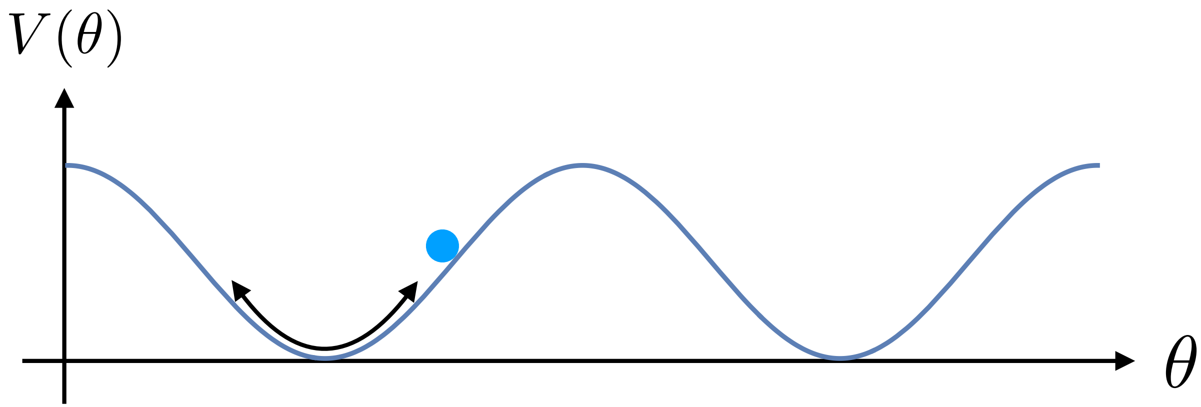
$$\left.\begin{aligned} \mathrm{B}_{2,3} &= 0.1 \\ \xi_\phi &= 200 \\ \phi_0 &= 10^{12}\mathrm{GeV} \\ Y_{\mathrm{PQ,e}} &= 10^{-6} \end{aligned}\right\}$$



- **Dark matter & dark radiation**

Dark matter

Cold axi-majorons from the misalignment only $\Rightarrow \phi_0 = \mathcal{O}(10^{12})$ GeV



Dark radiation

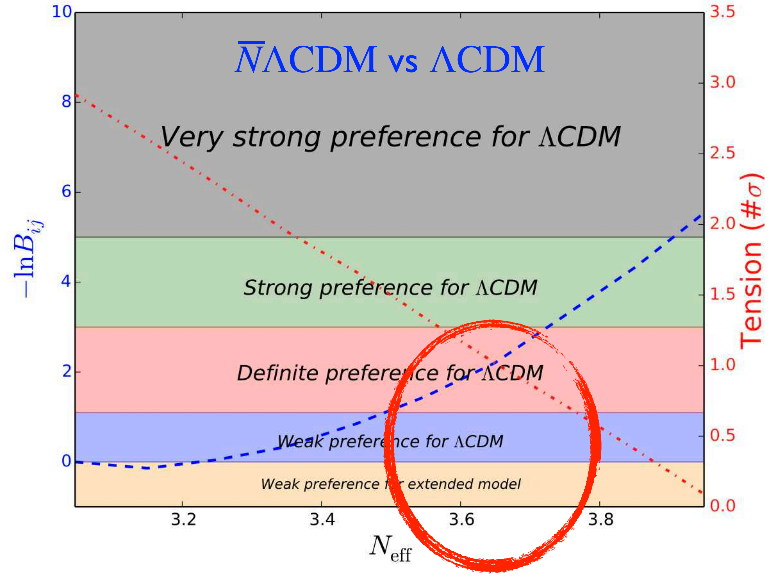
Hot axi-majorons from the decay of the inflaton:

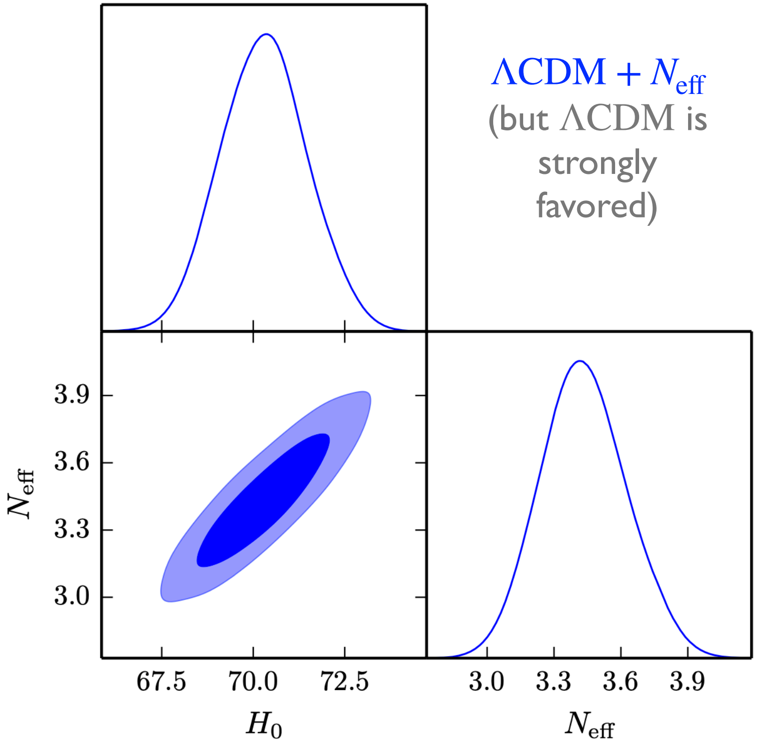
$$\Delta N_{\rm eff} \simeq 0.47 \left(\frac{10\Gamma_{N_1}}{H_{1,\rm eq}} \right)^{2/3}$$ $\Rightarrow$ partial alleviation of the Hubble tension

* Hubble tension

[Class. Quantum Grav. 38, 153001 (2021)]

< Impact of $\Delta N_{\rm eff}$ >





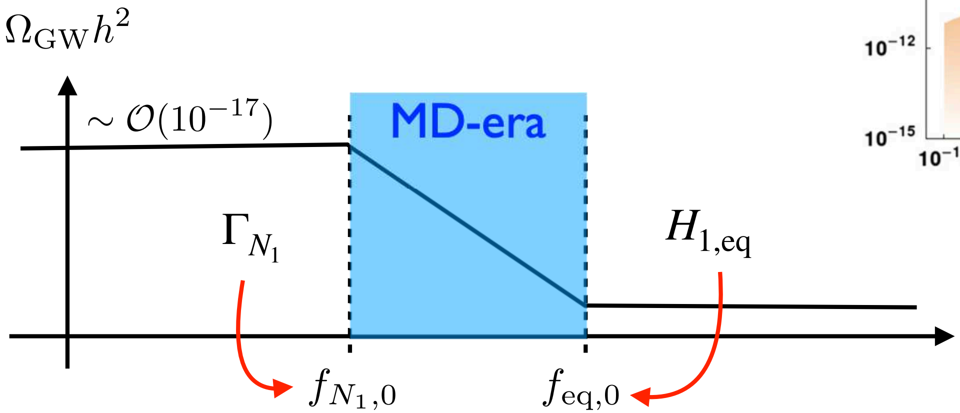
ΛCDM + $N_{\rm eff}$
(but ΛCDM is strongly favored)

[S.Vagnozzi, Phys.Rev.D 102 (2020) 2, 023518]

Signatures

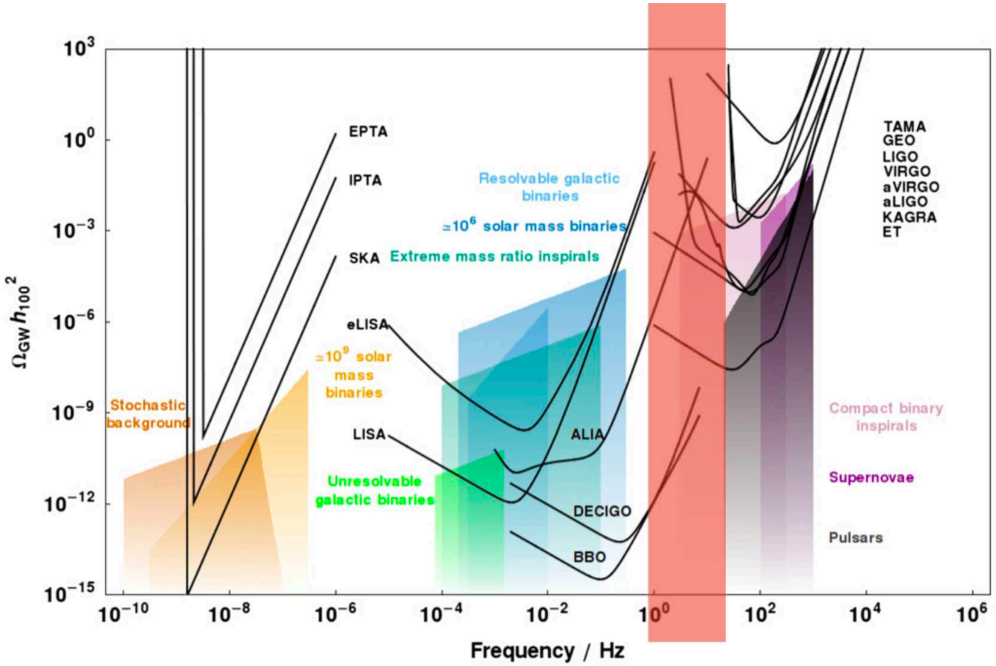
- **Distortion of inflationary GWs**

Characteristic frequencies:



$$f_{\rm eq,0} \approx 35 \times \left[\left(\frac{\rm B_1}{0.1}\right)^2 \left(\frac{100}{\xi_\phi}\right) \left(\frac{m_\nu}{10^4 m_{\nu_1}}\right) \right]^{2/3} f_{N_1,0}$$

$$f_{N_1,0} \approx 1{\rm Hz} \left[\left(\frac{0.1}{b_1}\right) \left(\frac{\xi_\phi}{100}\right)^2 \left(\frac{10^4 m_{\nu_1}}{m_\nu}\right) \left(\frac{\phi_0}{10^{12}{\rm GeV}}\right)^2 \right]^{1/2}$$



Consistency check:

$$\Delta N_{\rm eff} \simeq 0.48 \times \left(\frac{100}{g_*(T_{\gamma,\rm d})}\right)^{1/3} \left(\frac{10\Gamma_{N_1}}{H_{1,\rm eq}}\right)^{2/3}$$

$$({\rm c.f.}\ \ H_{\rm d} = (2/3)\Gamma_{N_1})$$

- **Presence of axi-Majoron DR**

$$\Delta N_{\rm eff} \simeq 0.48 \times \left(\frac{100}{g_*(T_{\gamma,\rm d})}\right)^{1/3} \left(\frac{10\Gamma_{N_1}}{H_{1,\rm eq}}\right)^{2/3}$$

$\Rightarrow$ Duration of MD-era

$\Rightarrow$ Change of the total inflationary e-folds(N_e)

- **Shift of $n_s(N_e)$ ($\Leftarrow$ Shift of N_e)**

$$n_s \simeq 1 - 2N_e^{-1}, \quad r_T \simeq 12N_e^{-2}$$

$$\Omega_{\rm GW} \xrightarrow{f_{N_1,0}} \Gamma_{N_1} \xrightarrow{\Delta N_{\rm eff}} H_{1,\rm eq} \xrightarrow[\Delta N_e]{\text{MD era}} n_s, r_T$$

- **Implication(s) on neutrinos physics**

$m_{\nu_1} \lll m_\nu \equiv 0.05\text{eV} \;\Rightarrow$ may have implications on observables of the neutrino pheno.

Summary

- We constructed the simplest minimal BSM model under the scale-sym. and $U(1)_{\mathrm{PQ}}$-sym.
- *$U(1)_{\mathrm{PQ}}$-breaking terms may exist only in the gravity sector.*
- *The model dubbed as the MCSM can address simultaneously following big puzzles:*

  - the origin of scales
  - the primordial inflation
  - the matter-antimatter asymmetry
  - dark matter and its relic density
  - dark radiation
  - the strong CP-problem
  - the origin of the tiny neutrino-mass
  - ~~the C.C. problem(maybe anthropic?)~~

  - Iso-curvature perturbations suppressed
  - No domain-wall problem
  - No axion-quality problem
  - No fifth force constraints
  - No hierarchy problem

- The MCSM provides the simplest unified framework for the history of the universe from inflation to the present universe (thanks to the father(Φ) and the mother(χ) of the universe)

Thank you!