기초과학연구원
Institute for Basic Science

IBS-CTPU-PTC

Lectures on Physics of Axions

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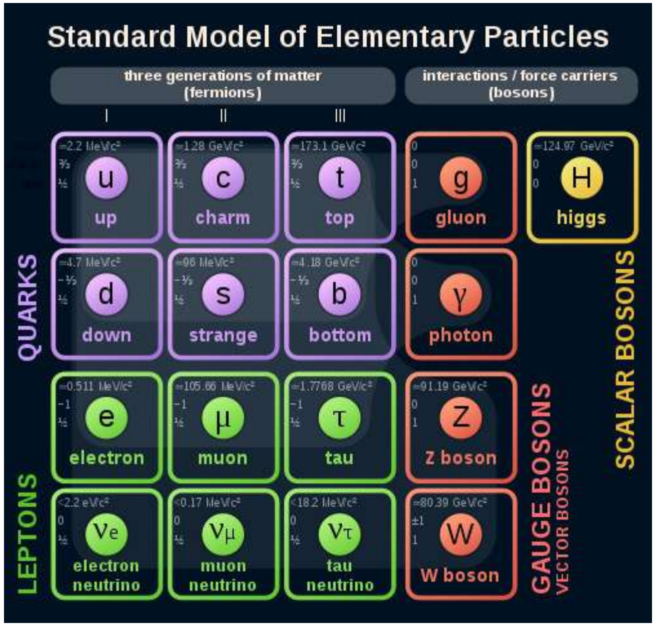
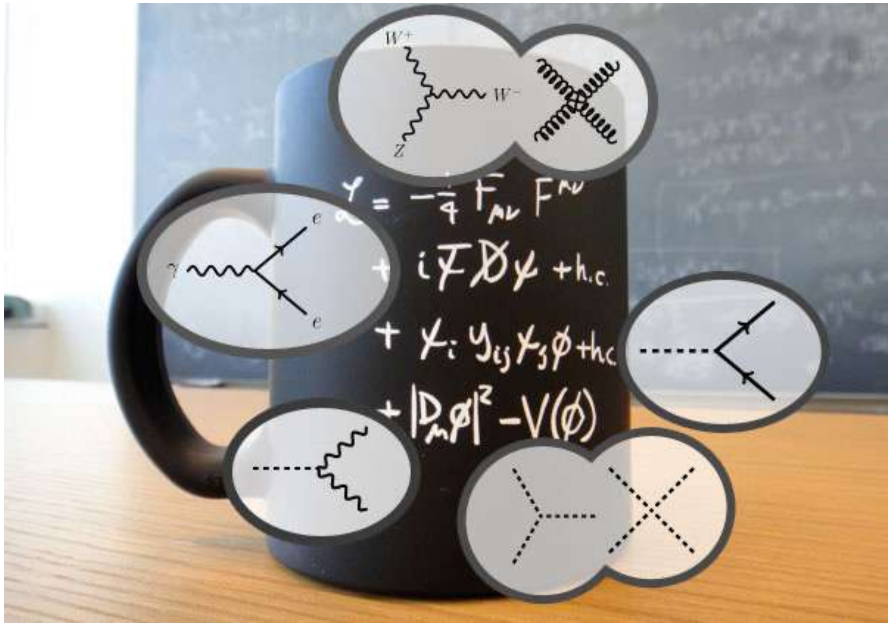
Outline

- Strong CP problem and QCD axion
- Axion-like particles (ALPs)
- Cosmology of axions: axion DM
- Experimental targets for QCD axion-photon coupling

The Standard Model (SM) of particle physics





- Consists of elementary particles and their interactions.
- Successfully describes almost all phenomena of nature occurring in a length scale longer than the Fermi scale ~ 0.01 fm.
- The SM lagrangian is invariant under Lorentz symmetry, CPT, and gauge symmetries $SU(3)_C \times SU(2)_L \times U(1)_Y$

Strong CP problem

The $SU(3)_c$ gauge symmetry & Lorentz symmetry allow

$$\mathcal{L}_{\rm SM} \supset \kappa\, \epsilon_{\mu\nu\rho\sigma} G^{a\mu\nu} G^{a\rho\sigma} \equiv 2\kappa\, G^{a\mu\nu} \widetilde{G}^a_{\mu\nu} \quad \left(\widetilde{G}^a_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{a\rho\sigma} \right)$$

$$\epsilon_{\mu\nu\rho\sigma} G^{\mu\nu} G^{\rho\sigma} = \underbrace{4\epsilon_{0ijk} G^{0i} G^{jk}}_{\vec{E}_g \cdot \vec{B}_g} \ (i,j,k = 1,2,3)$$

P and T(=CP) violation

P : $\vec{E}_g \to -\vec{E}_g,\ \vec{B}_g \to \vec{B}_g$

T (=CP) : $\vec{E}_g \to \vec{E}_g,\ \vec{B}_g \to -\vec{B}_g$

A non-zero κ implies a CP violation in the QCD sector.

Strong CP problem

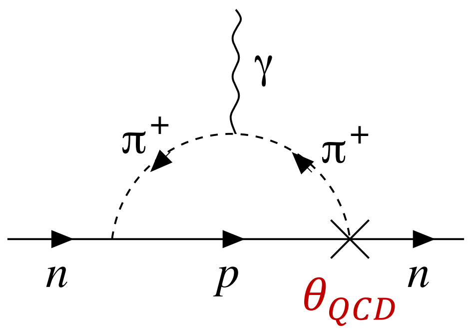
$$\mathcal{L}_{\rm SM} \supset \frac{g_3^2}{32\pi^2}\theta_{\rm QCD} G^{a\mu\nu}\widetilde{G}^a_{\mu\nu} \quad \left(\kappa \equiv \frac{g_3^2}{64\pi^2}\theta_{\rm QCD}\right)$$

QCD confinement

$$-d_N \frac{i}{2}\bar{N}\sigma^{\mu\nu}\gamma_5 N F_{\mu\nu} \ (N = p, n)$$

$$\simeq -2 d_N \vec{S}_N \cdot \vec{E}$$

Nucleon EDM operator (CP violating)



$$d_n \simeq 2.4 \times 10^{-16}\theta_{\rm QCD}\, e\, {\rm cm}$$: predicted neutron EDM

$$|d_n^{\rm exp}| < 1.8 \cdot 10^{-26} \ e \ {\rm cm} \ (90\% \ {\rm CL})$$

$$|\theta_{\rm QCD}| < 10^{-10}$$

But isn't it okay with simply assuming CP symmetry with $\theta_{QCD} = 0$?

Quark mass phase and strong CP

$$m_u u_L^\dagger u_R + \text{h.c.} = |m_u| \left(u_L^\dagger u_R e^{i\theta_u} + u_R^\dagger u_L e^{-i\theta_u} \right)$$

$$= |m_u| \bar{u}\, e^{i\theta_u \gamma^5} u \qquad\qquad u = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

$$m_u = |m_u| e^{i\theta_u} \qquad\qquad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

The quark mass phase may be eliminated by a field redefinition.

$$u \to e^{-i\theta_u \gamma^5 /2} u \quad \left(u_L \to e^{i\theta_u/2} u_L, \quad u_R \to e^{-i\theta_u/2} u_R \right)$$

"chiral" transformation

➡ $|m_u| \bar{u}\, e^{i\theta_u \gamma^5} u \;\to\; |m_u| \bar{u} u$

Quark mass phase and strong CP

The chiral transformation is "anomalous" quantum mechanically.

(= chiral anomaly)

$$Z = \int \mathcal{D}A\, \mathcal{D}\Psi\, \mathcal{D}\overline{\Psi}\ \exp\ i\int d^4x \left[i\overline{\Psi}\not{D}\Psi - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{g^2\theta}{32\pi^2}\tilde{F}^{a\mu\nu}F^a_{\mu\nu} \right]$$

$$\Psi \ \longrightarrow\ e^{-i\alpha\gamma_5}\,\Psi$$

➡ $$\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi} \rightarrow \exp\left[-i\int d^4x\ \frac{g^2\alpha}{16\pi^2}\tilde{F}^{a\mu\nu}F^a_{\mu\nu} \right] \mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}$$

The quantum measure is not invariant under the chiral transformation → introduces an additional θ-term.

Quark mass phase and strong CP

$$|m_u|\bar{u}\, e^{i\theta_u\gamma^5} u - \frac{g_3^2}{32\pi^2}\theta_{\rm QCD} G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}$$

$$u \to e^{-i\theta_u\gamma^5/2} u$$ ⬇

$$|m_u|\bar{u}u - \frac{g_3^2}{32\pi^2}\left(\theta_{\rm QCD} + \theta_u\right) G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}$$

$$\bar{\theta}_{\rm QCD} \equiv \theta_{\rm QCD} + \theta_u$$

turns out to be the true measurable strong CP phase due to the chiral anomaly.

Quark mass phase and strong CP

$$\bar{\theta}_{\rm QCD} \equiv \theta_{\rm QCD} + \underbrace{\theta_u}_{\mathrm{Arg}\, m_u}$$

multi-quark generalization $\rightarrow \mathrm{Arg}\ \det(M_q)$ $\left(M_{qij} q^\dagger_{Li} q_{Rj} + \mathrm{h.c.}\right)$

$i, j = u, d, s, \ldots$

$\rightarrow \mathrm{Arg}\ \det(y_u y_d)$

$\left(y_{uij} H^\dagger Q^\dagger_{Li} u_{Rj} + y_{dij} H Q^\dagger_{Li} d_{Rj} + \mathrm{h.c.}\right)$

$i, j = 1, 2, 3$

⬇

$$\bar{\theta}_{\rm QCD} = \theta_{\rm QCD} + \mathrm{Arg}\ \det(y_u y_d) < 10^{-10}$$

implies a severe fine tuning between a priori unrelated quantities in the SM → "strong CP problem"

: Note $\delta_{\rm CKM} = \mathrm{Arg}\ \det\left[y_u y_u^\dagger, y_d y_d^\dagger\right] \sim \mathcal{O}(1)$

Solution to the strong CP problem?

$$\bar{u}\gamma^\mu D_\mu u + |m_u|\bar{u}e^{i\theta_u\gamma^5}u - \frac{g_3^2}{32\pi^2}\theta_{\rm QCD}G^{a\mu\nu}\widetilde{G}^a_{\mu\nu} = \bar{u}\gamma^\mu D_\mu u - \frac{g_3^2}{32\pi^2}\theta_{\rm QCD}G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}$$

Massless u-quark?

$$u \to e^{-i\alpha\gamma^5/2}u$$

$$\bar{u}\gamma^\mu D_\mu u - \frac{g_3^2}{32\pi^2}(\theta_{\rm QCD}+\alpha)G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}$$

$\bar{\theta}_{QCD} = \theta_{QCD} + \alpha$ becomes an arbitrary parameter (i.e. physically irrelevant) and can be set to 0 by choosing $\alpha = -\theta_{QCD}$.

However, the massless u-quark solution is almost excluded by the current lattice QCD data (i.e. $m_u \neq 0$).

Digression : $\bar{\theta}_{QCD}$-dependent QCD vacuum energy

In the vacuum, $G_{\mu\nu} = 0$ $(\vec{E}_g = \vec{B}_g = 0)$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i g_3 [A_\mu, A_\nu]$$

⬇

$$A_\mu = \frac{i}{g_3} U_n(x) \partial_\mu U_n^+(x)$$: Not necessarily 0

$$U_n(x) = e^{i g_3 \Lambda_n^a(x) T^a}$$ T^a $(a = 1, \dots, 8)$: $SU(3)_c$ matrices

$U_n(x)$ is a periodic function over the spacetime and characterized by an integer number n (called "winding number") which counts how many times it periodically changes over the universe.

Therefore, the QCD vacuum is characterized by the winding number associated with the gluon field A_μ in the vacuum.

A vacuum with winding number n

$$A_\mu = \frac{i}{g_3} U_n(x)\partial_\mu U_n^+(x)$$

$$G_{\mu\nu} = 0$$

Smooth deformation of A_μ :

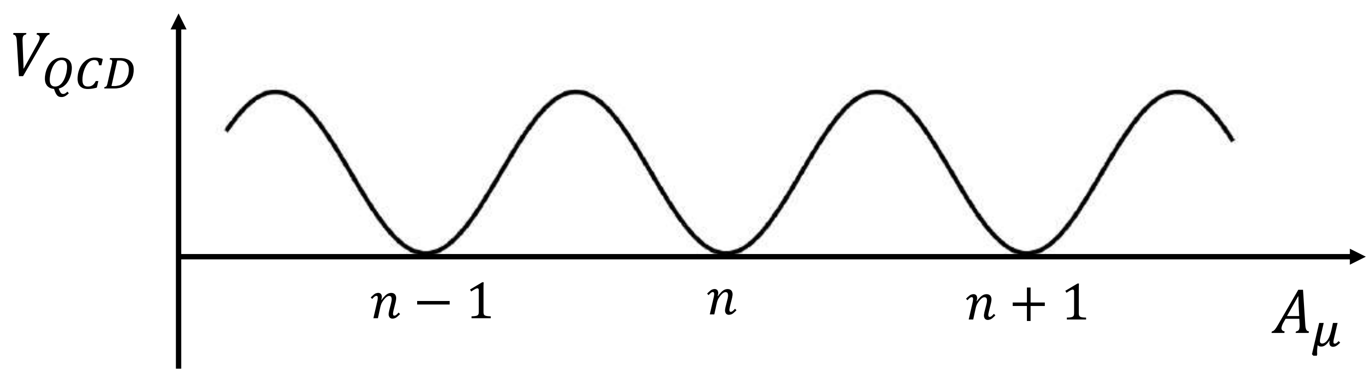
$$G_{\mu\nu} \neq 0$$

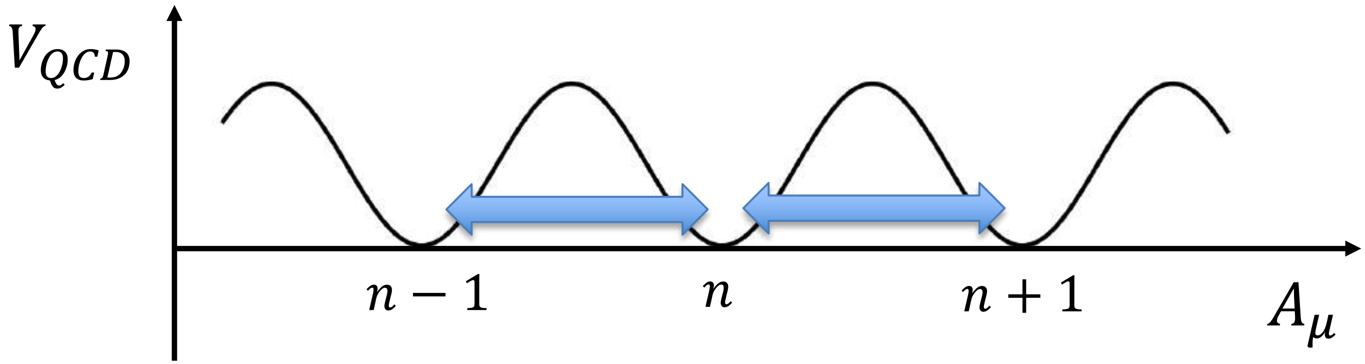
A vacuum with winding number $n+1$

$$A_\mu = \frac{i}{g_3} U_{n+1}(x)\partial_\mu U_{n+1}^+(x)$$

$$G_{\mu\nu} = 0$$

There is a potential barrier between the vacua with different winding numbers.





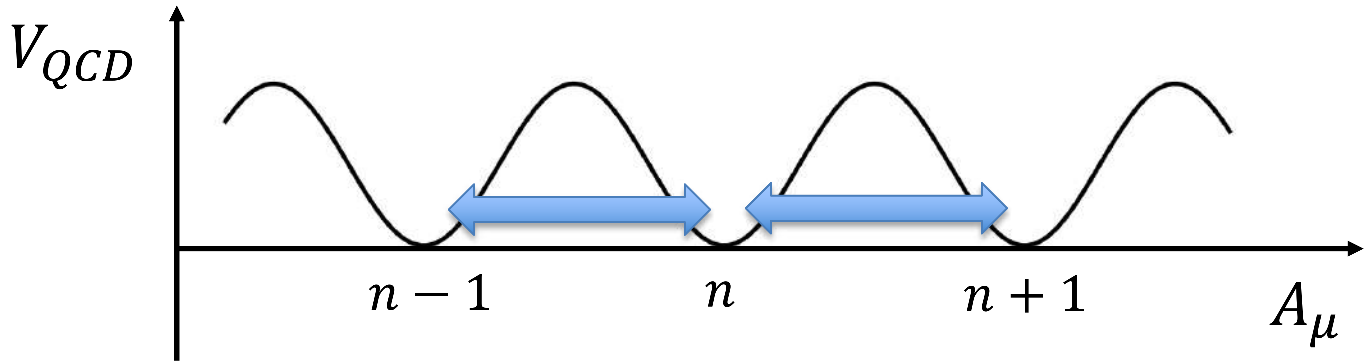
Quantum mechanically tunneling can happen between the vacua with different winding numbers.

The tunneling with $\Delta n = 1$ (-1) is called "(anti-)instanton process".

Because of the tunneling, the true vacuum state is given by a superposition of the classical vacuum states $|n\rangle$, which is dubbed "θ-vacuum".

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta} |n\rangle$$

($|\theta\rangle$ can be shown to be an eigenstate of any Hamiltonian of the form $\langle m|H|n\rangle = f(m-n)$.)



The θ-vacuum energy is non-zero due to the (anti-)instanton processes.

$$\langle\theta|H|\theta\rangle = \sum_{m,n}\langle m|e^{i(m-n)\theta}H|n\rangle \simeq \sum_n \langle n|H|n\rangle + \sum_n (\langle n+1|H|n\rangle e^{i\theta} + \langle n-1|H|n\rangle e^{-i\theta})$$

$$= \text{const} + \sum_n 2\langle n+1|H|n\rangle \cos\theta \;\sim \text{const} - \underbrace{V\Lambda_{QCD}^4\, e^{-S_{\text{ins}}}}\cos\theta$$

~ Tunneling amplitude due to the instanton and the anti-instanton

$$S_{\text{ins}} = \frac{8\pi^2}{g_3^2}$$: (Euclidean) instanton action

Λ_{QCD} : QCD confinement scale

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta}|n\rangle \quad \Rightarrow \quad V_{QCD}(\theta) \sim -\Lambda_{QCD}^4\, e^{-S_{\rm ins}} \cos\theta$$

But what is θ?

$$\langle\theta|e^{-iHt}|\theta\rangle = \sum_{m,n} \langle m|e^{-iHt}|n\rangle e^{i(m-n)\theta}$$

Computation from Hamiltonian formulation

$$\langle\theta|e^{-iHt}|\theta\rangle = \int [DA_\mu]_{\theta\leftarrow\theta} e^{i\,S} = \sum_{m,n} \int [DA_\mu]_{m\leftarrow n} e^{i\,S}$$

Path integral formulation

$$= \sum_{m,n} \int [DA_\mu]_{m\leftarrow n} e^{i\,S_{\rm kin} + i(m-n)\bar{\theta}_{QCD}} = \sum_{m,n} \langle m|e^{-iHt}|n\rangle e^{i(m-n)\bar{\theta}_{QCD}}$$

where $S = \int d^4x\, (-\frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{g_3^2}{32\pi^2}\, \bar{\theta}_{QCD}\, G^{a\mu\nu} \tilde{G}^a_{\mu\nu})$

Thus $\theta = \bar{\theta}_{QCD}$

So we have shown that the (anti-)instanton processes yield vacuum energy which depends on $\bar{\theta}_{QCD}$.

$$V \approx -\Lambda_{\rm QCD}^4 e^{-S_{\rm ins}} \cos\bar{\theta}_{\rm QCD}$$

("non-perturbative" ; cannot be described in terms of g_3^2-expansion)

$$S_{\rm ins} = \frac{8\pi^2}{g_3^2}$$: instanton action

Λ_{QCD} : confinement energy of QCD

This suggests the "axion solution" to the strong CP problem.

$$\bar{\theta}_{\rm QCD} \;\rightarrow\; \bar{\theta}_{\rm QCD,eff} = \bar{\theta}_{\rm QCD} + \frac{a(x)}{f_a}$$

$$\Rightarrow \quad V \approx -\Lambda_{\rm QCD}^4 e^{-S_{\rm ins}} \cos\left(\bar{\theta}_{\rm QCD} + \frac{a(x)}{f_a}\right)$$

The field $a(x)$ dynamics will minimize the potential so that $\frac{\langle a(x)\rangle}{f_a} = -\bar{\theta}_{\rm QCD}$

$$\Rightarrow \quad \bar{\theta}_{\rm QCD,eff} = 0$$

Axion solution to the strong CP problem

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{g_3^2}{32\pi^2}\left(\bar{\theta}_{\rm QCD} + \frac{a}{f_a}\right) G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}$$

This lagrangian cannot be fundamental, since it is non-renormalizable due to the dim-5 operator.

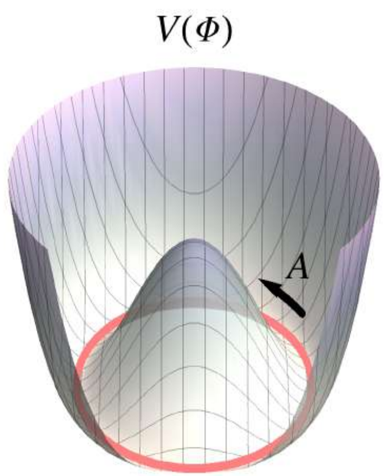
What's the UV completion of the above lagrangian ?

- PQWW model (ruled out)
- KSVZ model
- DFSZ model
- String theoretic QCD axion model

UV model for axion

Consider a complex scalar field $\Phi(\mathrm{x})$

$$\mathcal{L} = |\partial_\mu \Phi|^2 - V(\Phi)$$



$$V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4 \qquad \mu^2 > 0$$

The lagrangian has a global symmetry :

$$U(1)_{\mathrm{PQ}} : \Phi \;\to\; \Phi\, e^{i\alpha}$$

"Peccei-Quinn symmetry"

$$\Phi(x) = \frac{1}{\sqrt{2}}\rho(x)e^{i\theta(x)}$$

$\rho(x)$: non-compact real scalar field

$\theta(x)$: compact (i.e. periodic) real scalar field

$$U(1)_{\mathrm{PQ}} : (\Phi \rightarrow \Phi\, e^{i\alpha}) \iff (\theta(x) \rightarrow \theta(x) + \alpha)$$

: Shift symmetry of $\theta(x)$

$$\begin{aligned}\mathcal{L} &= |\partial_\mu \Phi|^2 - V(\Phi) \\ &= \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}\rho^2(\partial_\mu \theta)^2 - V(\rho)\end{aligned}$$

$$\begin{aligned}V(\rho) &= -\mu^2|\Phi|^2 + \lambda|\Phi|^4 \\ &= -\frac{\mu^2}{2}\rho^2 + \frac{\lambda}{4}\rho^4\end{aligned}$$

$$\frac{\partial V(\rho)}{\partial \rho} = 0 \quad \Rightarrow \quad \langle \rho \rangle = \frac{\mu}{\sqrt{\lambda}} \equiv f_a$$

$$\rho = \langle\rho\rangle + \tilde{\rho} = f_a + \tilde{\rho} \qquad \theta(x) \equiv \frac{a(x)}{f_a}$$

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\rho^2(\partial_\mu\theta)^2 + \frac{1}{2}(\partial_\mu\rho)^2 - V(\rho) \\ =& \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu\tilde{\rho})^2 - \frac{1}{2}m_\rho^2\tilde{\rho}^2 - \lambda f_a\tilde{\rho}^3 - \frac{\lambda}{4}\tilde{\rho}^4 + \frac{\lambda}{4}f_a^4 \\ &+ \frac{1}{f_a}\tilde{\rho}(\partial_\mu a)^2 + \frac{1}{2f_a^2}\tilde{\rho}^2(\partial_\mu a)^2\end{aligned}$$

$\Rightarrow$ $m_a^2 = 0, \quad m_\rho^2 = 2\lambda f_a^2$

The axion field $a(x)$ is massless because of the PQ symmetry:

$$U(1)_{\rm PQ} : (\theta(x) \to \theta(x) + \alpha) \iff (a(x) \to a(x) + f_a\alpha)$$

Now suppose a Yukawa coupling between the PQ complex scalar and a quark :

$$\mathcal{L} = \bar{q} i \gamma^\mu D_\mu q + \left(y\, \Phi q_R^\dagger q_L + \text{h.c.} \right) + \frac{g_3^2}{32\pi^2} \bar{\theta}_{QCD} G^{a\mu\nu} \widetilde{G}^a_{\mu\nu}$$

$$y\, \Phi q_R^\dagger q_L = y \frac{1}{\sqrt{2}} (f_a + \tilde{\rho}) e^{ia(x)/f_a} q_R^\dagger q_L$$

Chiral transformation (axion-dependent field redefinition)

$$q_L \to e^{-ia(x)/2f_a} q_L, \quad q_R \to e^{ia(x)/2f_a} q_R$$

$$\left(q \to e^{ia(x)\gamma^5/2f_a} q \right)$$

$$\mathcal{L} = \bar{q} i \gamma^\mu D_\mu q + \left(\frac{y}{\sqrt{2}} (f_a + \tilde{\rho}) q_R^\dagger q_L + \text{h.c.} \right) - \frac{\partial_\mu a}{2 f_a} \bar{q} \gamma^\mu \gamma^5 q + \frac{g_3^2}{32\pi^2} \left(\bar{\theta}_{QCD} + \frac{a(x)}{f_a} \right) G^{a\mu\nu} \widetilde{G}^a_{\mu\nu}$$

- Quark mass: $\frac{y}{\sqrt{2}} (f_a + \tilde{\rho}) q_R^\dagger q_L$
- contributes no potential for $a(x)$: $- \frac{\partial_\mu a}{2 f_a} \bar{q} \gamma^\mu \gamma^5 q$
- desired term for the strong CP problem: $\left(\bar{\theta}_{QCD} + \frac{a(x)}{f_a} \right)$

$$\mathcal{L} = \bar{q} i \gamma^\mu D_\mu q + \left(\frac{y}{\sqrt{2}} (f_a + \tilde{\rho}) q_R^\dagger q_L + \text{h.c.} \right) - \frac{\partial_\mu a}{2 f_a} \bar{q} \gamma^\mu \gamma^5 q + \frac{g_3^2}{32\pi^2} \left(\bar{\theta}_{QCD} + \frac{a(x)}{f_a} \right) G^{a\mu\nu} \widetilde{G}^a_{\mu\nu}$$

The lagrangian is invariant under $U(1)_{PQ}$: $a(x) \rightarrow a(x) + f_a \alpha$ *perturbatively*.
This symmetry is broken only by the QCD instanton effect *non-perturbatively*.
So the axion has a potential only from the QCD instanton effect.

$$V(a) \simeq -\Lambda_{\rm QCD}^4 e^{-8\pi^2/g_3^2} \cos\left(\frac{a}{f_a} + \bar{\theta}_{\rm QCD} \right)$$

In order to solve the strong CP problem, $U(1)_{PQ}$ has to be a good symmetry of the theory up to the QCD instanton effect :

$$V(a) = -\Lambda_{\rm QCD}^4 e^{-8\pi^2/g_3^2} \cos\left(\frac{a}{f_a} + \bar{\theta}_{\rm QCD} \right) + \Delta V(a)$$

$$\frac{\langle a \rangle}{f_a} + \bar{\theta}_{\rm QCD} \neq 0 \quad \text{for } \Delta V(a) \neq 0$$

$\Delta V(a)$ has to be very small so that the strong CP angle $\bar{\theta}_{QCD,eff} = \frac{\langle a \rangle}{f_a} + \bar{\theta}_{QCD} < 10^{-10}$.

$$\mathcal{L} = \bar{q} i \gamma^\mu D_\mu q + \left(y\, \Phi q_R^\dagger q_L + \text{h.c.} \right) + \left(M_q q_R^\dagger q_L + \text{h.c.} \right) + \frac{g_3^2}{32\pi^2} \bar{\theta}_{QCD} G^{a\mu\nu} \widetilde{G}^a_{\mu\nu}$$

$U(1)_{PQ}$ invariant

$U(1)_{PQ}$ breaking $\rightarrow \Delta V(a)$

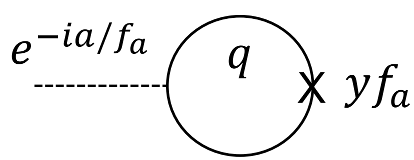
$$U(1)_{\rm PQ} \begin{cases} \Phi \rightarrow \Phi e^{i\alpha} \quad \left(= \dfrac{a(x)}{f_a} \rightarrow \dfrac{a(x)}{f_a} + \alpha \right) \\ q_L \rightarrow q_L e^{-i\alpha/2} \\ q_R \rightarrow q_R e^{i\alpha/2} \end{cases}$$

$$q_L \rightarrow e^{-ia(x)/2f_a} q_L, \quad q_R \rightarrow e^{ia(x)/2f_a} q_R$$

$$\mathcal{L} = \bar{q} i \gamma^\mu D_\mu q + \left(\frac{y}{\sqrt{2}} (f_a + \tilde{\rho}) q_R^\dagger q_L + \text{h.c.} \right) - \frac{\partial_\mu a}{2 f_a} \bar{q} \gamma^\mu \gamma^5 q + \frac{g_3^2}{32\pi^2} \left(\bar{\theta}_{QCD} + \frac{a(x)}{f_a} \right) G^{a\mu\nu} \widetilde{G}^a_{\mu\nu}$$

$$+ \left(M_q e^{-ia/f_a} q_R^\dagger q_L + \text{h.c.} \right)$$

$$\Delta V(a) \sim \frac{y f_a}{16\pi^2} M_q \Lambda^2 \cos\frac{a}{f_a}$$



$$\mathcal{L} = \bar{q} i\gamma^\mu D_\mu q + \left(y\,\Phi q_R^\dagger q_L + \text{h.c.}\right) + \left(M_q q_R^\dagger q_L + \text{h.c.}\right) + \frac{g_3^2}{32\pi^2}\bar{\theta}_{QCD} G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}$$

$$U(1)_{\rm PQ} \begin{cases} \Phi \to \Phi e^{i\alpha} \quad \left(= \dfrac{a(x)}{f_a} \to \dfrac{a(x)}{f_a} + \alpha\right) \\ q_L \to q_L e^{-i\alpha\cdot 1/2} \\ q_R \to q_R e^{i\alpha\cdot 1/2} \end{cases}$$

$$q \to q e^{i\gamma^5\alpha\cdot\frac{1}{2}}$$

Here ½ = PQ charge of the quark

$$q_L \to e^{-ia(x)/f_a\cdot 1/2} q_L, \quad q_R \to e^{ia(x)/f_a\cdot 1/2} q_R$$

$$\mathcal{L} = \bar{q} i\gamma^\mu D_\mu q + \left(\frac{y}{\sqrt{2}}(f_a + \tilde{\rho}) q_R^\dagger q_L + \text{h.c.}\right) - \frac{1}{2}\frac{\partial_\mu a}{f_a}\bar{q}\gamma^\mu\gamma^5 q + \frac{g_3^2}{32\pi^2}\left(\bar{\theta}_{QCD} + 1\cdot\frac{a(x)}{f_a}\right) G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}$$

$$+ \left(M_q e^{-ia/f_a} q_R^\dagger q_L + \text{h.c.}\right)$$

quark PQ charge

2 × quark PQ charge

$$U(1)_{\rm PQ} \begin{cases} \dfrac{a(x)}{f_a} \to \dfrac{a(x)}{f_a} + \alpha \\ q_L \to q_L \\ q_R \to q_R \end{cases}$$

In the redefined field basis, only the axion field transforms under $U(1)_{PQ}$.

Georgi-Kaplan-Randall basis ‘86

Peccei-Quinn-Weinberg-Wilczek (PQWW) model

[Peccei, Quinn '77, Weinberg '78, Wilczek '78]

$$y\,\Phi q_R^\dagger q_L$$

q : SM quarks

Φ : another Higgs doublet

SM : $y_u u_R^\dagger Q_L H + y_d d_R^\dagger Q_L H^*$ $\quad H(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \underbrace{e^{i\alpha^a(x)\sigma^a/2}}$

1 in the unitary gauge; No axion

2 Higgs doublet extension : $y_u u_R^\dagger Q_L H_u + y_d d_R^\dagger Q_L H_d \quad v^2 = v_u^2 + v_d^2$

$$H_u(x) \approx H_{\rm SM}(x) \qquad H_d(x) \approx \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + H_0(x))e^{ia(x)/v_d} \\ H^\pm(x) \end{pmatrix}$$

In the limit $v_u \gg v_d$

$f_a \lesssim v$: excluded by collider experiments

Kim-Shiftman-Vainshtein-Zakharov (KSVZ) model

[Kim '79, Shifman, Vainshtein, Zakharov '80]

$$y\,\Phi q_R^\dagger q_L$$

q : exotic heavy quark
Φ : SM gauge-singlet complex scalar

$$m_q = \frac{y}{\sqrt{2}} f_a$$

f_a can be much larger than the weak scale.
→ tiny axion coupling (invisible axion)

The KSVZ model predicts a new heavy quark unless the Yukawa coupling y is unreasonably small.

Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) model

[Dine, Fischler, Srednicki '81, Zhitnitsky '80]

$m_q\, e^{-ia(x)/f_a} q_R^\dagger q_L$ $\quad q$: SM quarks

$$y_u u_R^\dagger Q_L H_u + y_d d_R^\dagger Q_L H_d + \lambda_{\Phi H} \Phi^2 H_u H_d \qquad \Phi = \frac{1}{\sqrt{2}}(f_a + \tilde{\rho}) e^{ia(x)/f_a}$$

$$H_d \rightarrow H_d e^{-2ia(x)/f_a}$$

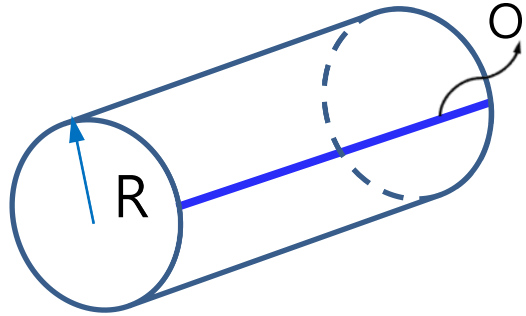
$$y_u u_R^\dagger Q_L H_u + y_d e^{-2ia/f_a} d_R^\dagger Q_L H_d + \frac{\lambda_{\Phi H}}{2}(f_a + \tilde{\rho})^2 H_u H_d$$

Here f_a can be large in contrast to the PQWW model.
→ Another realization of an invisible axion

String theoretic QCD axion

Witten '84

Simplified 5D example :



Compact extra dimension

$M, N = 0, 1, 2, 3, 5$

$$S = \int d^4x dx_5 \left(-\frac{1}{4g_5^2} F^{MN} F_{MN} + \kappa \, \epsilon^{MNPQR} A_M G^a_{NP} G^a_{QR} \right)$$

$$= \int d^4x \left(-\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} (\partial_\mu a)^2 + \underbrace{2\pi R \, g\kappa \, a \, \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}} + \cdots \right)$$

$$a(x^\mu) = \frac{1}{2\pi R} \oint dx_5 A_5(x^\mu, x_5)$$

$$= \frac{1}{32\pi^2} \frac{a}{f_a} G^{a\mu\nu} \widetilde{G}^a_{\mu\nu}$$

4D axion identified as a higher dimensional component of the gauge field

$$f_a \sim \frac{1}{16\pi^2} R^{-1}$$

QCD Axion couplings

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{\partial_\mu a}{f_a}\sum_q c_q \bar{q}\gamma^\mu\gamma^5 q + \frac{1}{32\pi^2} c_g \frac{a}{f_a} G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}$$
$$+ \frac{\partial_\mu a}{f_a}\left(\sum_\ell c_\ell \bar{\ell}\gamma^\mu\gamma^5 \ell + c_H H^\dagger i \overleftrightarrow{D}^\mu H\right) + \frac{1}{32\pi^2}\frac{a}{f_a}\left(c_W W^a_{\mu\nu}\widetilde{W}^{a\mu\nu} + c_B B_{\mu\nu}\widetilde{B}^{\mu\nu}\right)$$

c_g is the necessary coupling to solve the strong CP problem, and the other couplings depend on specific UV models.

Phenomenological constraints on f_a from the axion-gluon coupling:

$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$$ "QCD axion window"

Star cooling from axion emission

Axion DM overproduction

QCD Axion mass

When $T \gg \Lambda_{QCD}$, (T: temperature of the early Universe)

$$\frac{1}{32\pi^2}\left(\frac{a}{f_a}+\bar{\theta}_{\rm QCD}\right)G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}$$

QCD instanton

$$\Rightarrow \quad V(a) \simeq -T^4 e^{-8\pi^2/g_3^2(T)}\cos\left(\frac{a}{f_a}+\bar{\theta}_{\rm QCD}\right)$$

$$\Rightarrow \quad m_a^2 = \partial_a^2 V(a)|_{\rm min} \simeq \frac{T^4}{f_a^2}e^{-8\pi^2/g_3^2(T)}$$

temperature-dependent
axion mass

QCD Axion mass

When $T < \Lambda_{QCD}$, the QCD confinement happens, and one has to take into account quark confinement effect.

$$m_u u_R^\dagger u_L + \text{h.c.} + \frac{1}{32\pi^2}\left(\frac{a}{f_a} + \bar{\theta}_{\text{QCD}}\right) G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}$$

$$\Rightarrow \quad V(a,\eta') \approx -m_u \Lambda^3_{\text{QCD}} \cos\frac{\eta'}{f_{\eta'}} - \Lambda^4_{\text{QCD}} \cos\left(\frac{\eta'}{f_{\eta'}} + \frac{a}{f_a} + \bar{\theta}_{\text{QCD}}\right)$$

$$u_R^\dagger u_L \Rightarrow \Lambda^3_{\text{QCD}} e^{i\eta'/f_{\eta'}} \qquad \approx -m_u \Lambda^3_{\text{QCD}} \cos\left(\frac{a}{f_a} + \bar{\theta}_{\text{QCD}}\right)$$

$$\partial_{\eta'} V(a,\eta') = 0$$

$$\Rightarrow \quad m_a^2 \sim \frac{m_u \Lambda^3_{\text{QCD}}}{f_a^2}$$

QCD Axion mass

More precisely including the other light quarks u, d, s (using the chiral perturbation theory),

$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2 f_a}\right)}$$

➡ $$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

➡ $$\boxed{m_a \simeq 5.7 \left(\frac{10^{12}\ \mathrm{GeV}}{f_a}\right) \mu\mathrm{eV}}$$

Axion-Like Particle (ALP)

Theoretically similar to the QCD axion, not being involved in a solution to the strong CP problem.

i) Periodicity : $a(x) \rightarrow a(x) + 2\pi n f_a$ $\quad$ ($\frac{a(x)}{f_a} = \theta(x)$; angle)

ii) Approximate shift symmetry $U(1)_{PQ}$: $a(x) \rightarrow a(x) + c$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{\partial_\mu a}{f_a}\left(c_\psi \bar{\psi}\gamma^\mu \psi + c_H H^\dagger i \overleftrightarrow{D}^\mu H\right) + \frac{c_A}{32\pi^2}\frac{a}{f_a}F^A_{\mu\nu}\widetilde{F}^{A\mu\nu}$$

i) → Natural size of ALP couplings is determined by f_a.

ii) → ALP is naturally light.

ALP mass by nonperturbative effect

$$\frac{1}{32\pi^2}\frac{a}{f_a}G_H\widetilde{G}_H \quad \Rightarrow \quad V(a) \approx -\Lambda_H^4 e^{-8\pi^2/g_H^2}\cos\frac{a}{f_a}$$

Λ_H : confining scale of the hidden gluons

$$\Rightarrow \quad m_a^2 \approx \frac{\Lambda_H^4}{f_a^2}e^{-8\pi^2/g_H^2}$$

The axion mass m_a and coupling $1/f_a$ are basically free parameters with theoretical naturalness by the symmetries.

Why ALPs ?

- QCD axion : if it really exists, probably not alone.
- String theory : generally many 4D ALPs from higher dimensional gauge fields upon compactification of extra dimensions
- The conventional BSM scenarios (e.g. GUT, SUSY, composite Higgs) involve ALPs in many cases in different context.

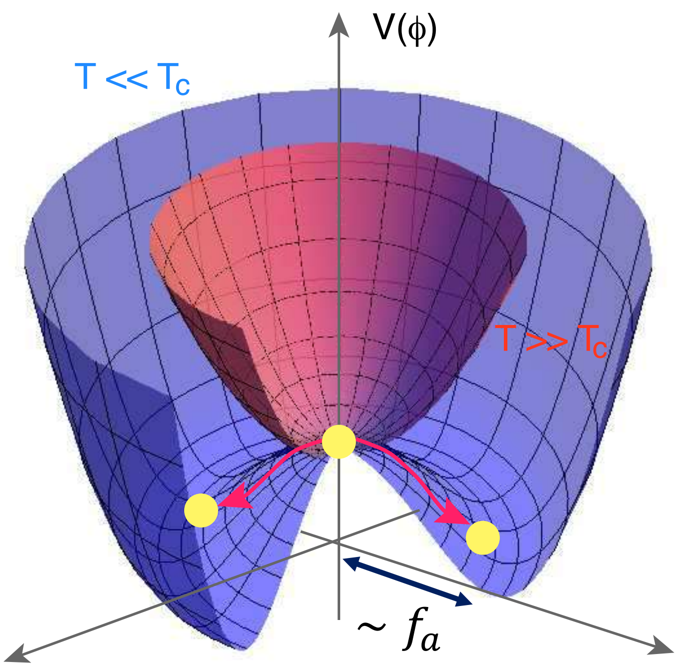
Cosmology of axions : axion dark matter

Spontaneous PQ breaking in the early universe

$$\mathcal{L} = |\partial_\mu \Phi|^2 - V(\Phi) \qquad U(1)_{\rm PQ} : \Phi \to \Phi\, e^{i\alpha} \qquad y\,\Phi q_R^\dagger q_L$$

$$V(\Phi) = \left(\kappa T^2 - \mu^2\right)|\Phi|^2 + \lambda|\Phi|^4 \qquad \kappa = \frac{3}{2}\lambda + \cdots \quad \sim y^2$$

: thermal mass from Φ's interaction with thermal plasma



$$T_c = \frac{\mu}{\sqrt{\kappa}} \sim f_a$$

$T > T_c$: $\langle \Phi \rangle = 0$
(no massless field → no axion)

$T \ll T_c$: $\langle \Phi \rangle = \frac{\lambda}{\sqrt{2\mu}} \equiv \frac{f_a}{\sqrt{2}}$
($U(1)_{PQ}$ spontaneously broken → a massless goldstone boson = axion)

Axion cosmology is very different depending on when the spontaneous PQ breaking happens

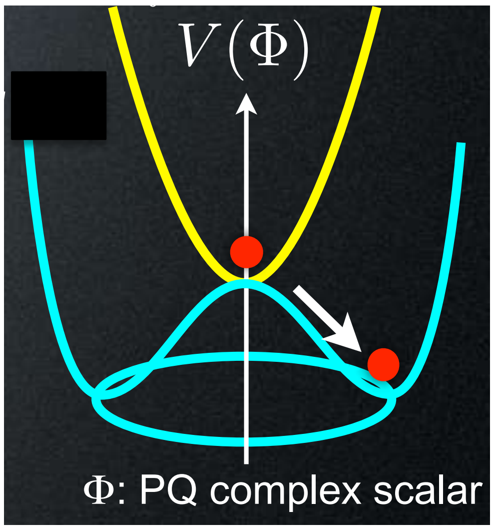
i) *before/during* inflation (and never be restored afterward)

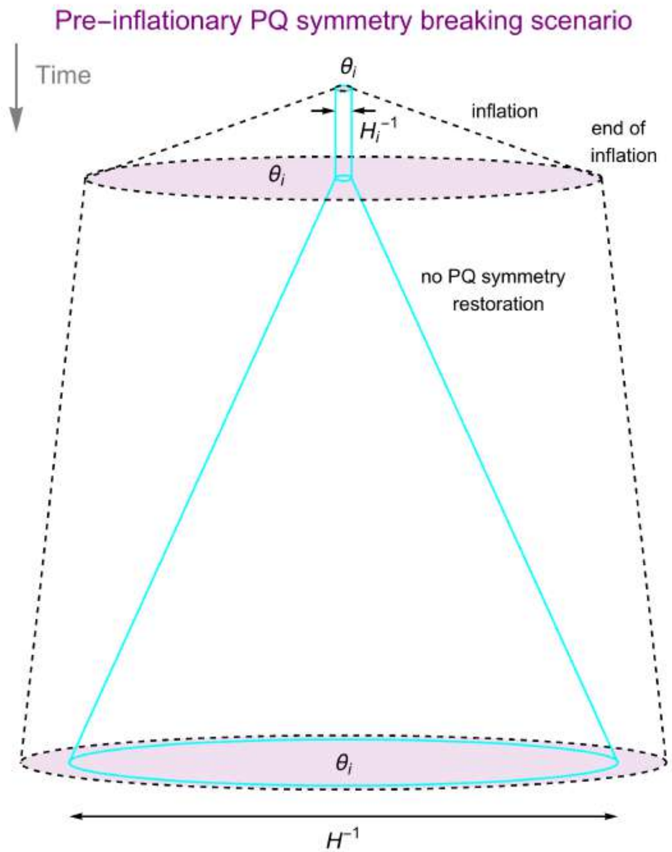
: Gibbons-Hawking temperature $T_I = \frac{H_I}{2\pi} \lesssim f_a$

ii) *after* inflation

: Reheating temperature (temperature right after inflation) $T_R \gtrsim f_a$

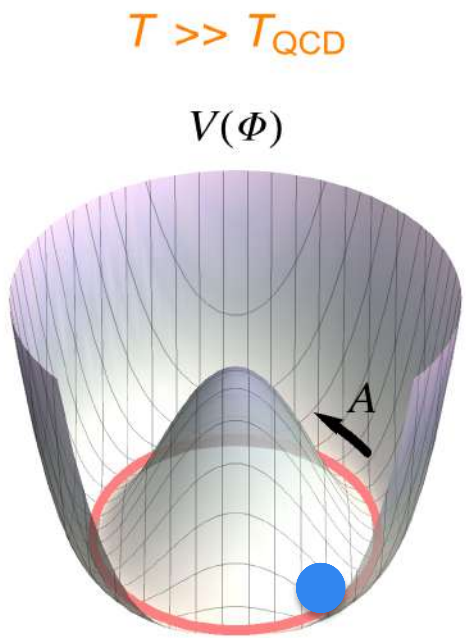
i) PQ spontaneously broken before/during inflation



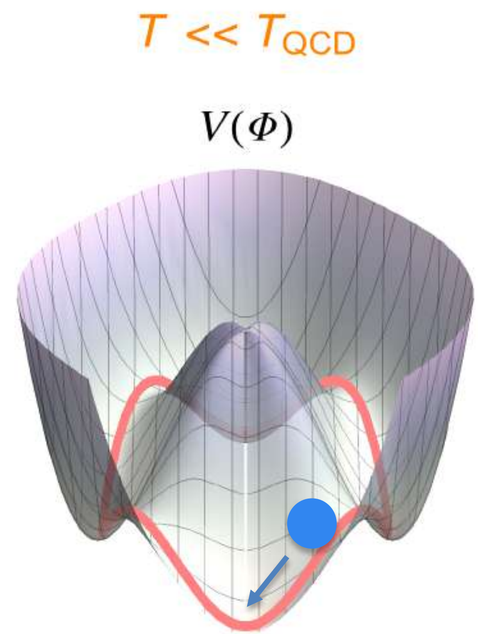


The inflationary expansion makes the axion field value universal over the observable patch of the universe.

Axion field dynamics in the pre-inflationary PQ breaking scenario



Same initial axion field value over the universe

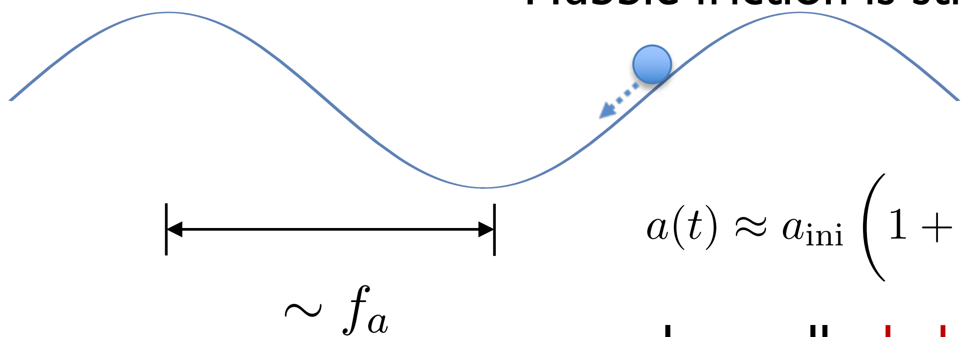


The axion field starts to move after its potential is developed.

Misalignment production of axion dark matter

Hubble friction is strong

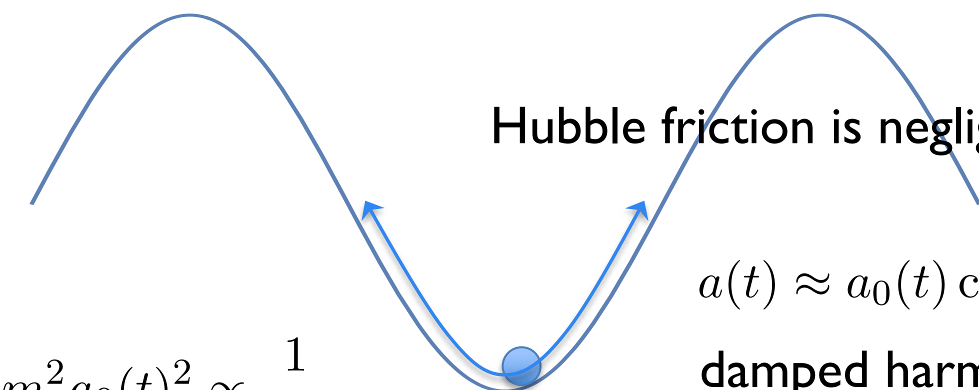
$$m_a \ll H(t)$$



$$a(t) \approx a_{\rm ini}\left(1 + \frac{1}{20}\left[\frac{m_a^2}{H_{\rm ini}^2} - \frac{m_a^2}{H(t)^2}\right]\right)$$

slow-roll : dark energy

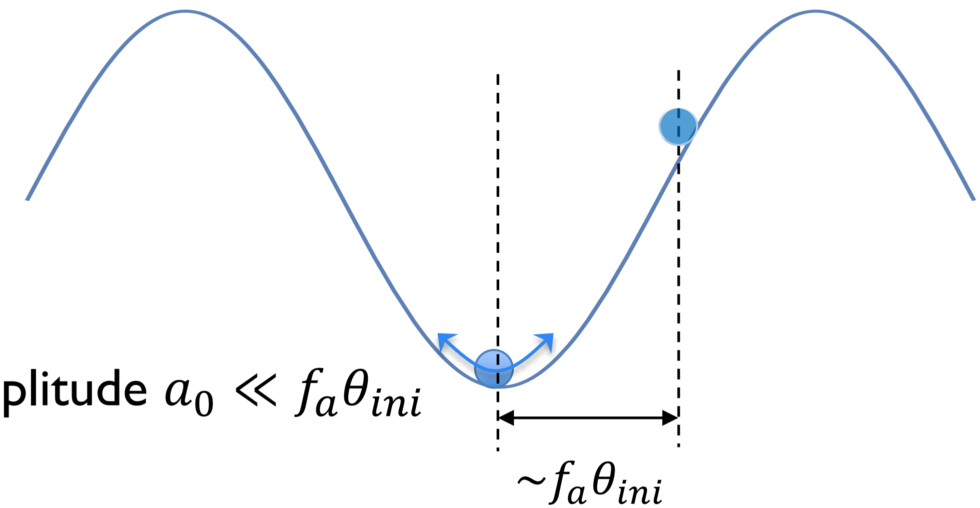
$$m_a \gg H(t)$$



$$a(t) \approx a_0(t)\cos(m_a t)$$

$$\rho_a \approx \frac{1}{2} m_a^2 a_0(t)^2 \propto \frac{1}{R^3}$$

damped harmonic oscillator : cold dark matter



Present oscillation amplitude $a_0 \ll f_a\theta_{ini}$

In the present universe, the oscillation amplitude is significantly smaller than the initial amplitude ($\sim f_a$), because it has been damped.

$$a(t,\vec{x}) \approx a_0 \cos m_a(t - \vec{v}\cdot\vec{x})$$

$|\vec{v}| \sim 10^{-3}$: virial velocity of the axion DM

$$\rho_a \approx \frac{1}{2} m_a^2 a_0^2 \quad \Rightarrow \quad \frac{\rho_a}{\rho_{\rm DM}} \simeq \sqrt{\frac{m_a}{1\,\mu{\rm eV}}}\left(\frac{f_a\theta_{\rm ini}}{10^{13}\,{\rm GeV}}\right)^2 \times \underbrace{\sqrt{\frac{m_a}{m_{a,{\rm osc}}}}}_{O(100)}$$

$O(100)$ for QCD axion

The present amplitude a_0 is determined from the initial amplitude $\sim f_a\theta_{ini}$ and the axion mass m_a & $m_{a,osc}$.

$$V(a) = -\Lambda_a^4(T)\cos\frac{a}{f_a} \approx V_0 + \frac{1}{2}m_a^2(T)a^2 \qquad m_a^2(T) = \frac{\Lambda_a^4(T)}{f_a^2}$$

Eq. of motion $\quad \ddot{a} + 3H(t)\dot{a} + m_a^2(t)a = 0 \qquad H(t) = \frac{\dot{R}(t)}{R(t)}$

Solution ansatz $\quad a(t) = A(t)\cos\left(\int_{t_1}^{t} dt' m_a(t')\right)$

$H(t) \neq 0$ (WKB approx.) $\qquad m_a(t) \gg H(t)$

$$A(t) \approx A_1\sqrt{\frac{m_1 R_1^3}{m_a(t)R^3(t)}}$$

Sol. ($m_a(t) \gg H(t)$)

Subscript "1" : initial value (at the oscillation start time)

Energy-momentum tensor

$$\rho \;=\; \frac{1}{2}\dot{a}^2 + \frac{1}{2}m_a^2 a^2 = \frac{1}{2}m_a^2 A^2\left[1 + \mathcal{O}\left(\frac{H}{m_a}\right)\right]$$

$$p \;=\; \frac{1}{2}\dot{a}^2 - \frac{1}{2}m_a^2 a^2 = -\frac{1}{2}m_a^2 A^2 \cos\left(2\int_{t_1}^{t} dt' m_a\right)\left[1 + \mathcal{O}\left(\frac{H}{m_a}\right)\right]$$

$$A(t) \approx A_1\sqrt{\frac{m_1 R_1^3}{m_a(t) R^3(t)}}$$

$$\rho \approx \frac{1}{2}m_a^2 A^2 \approx \frac{1}{2}m_a m_1 A_1^2 \frac{R_1^3}{R^3} \propto \frac{1}{R^3}$$ as $m_a(t) \rightarrow$ const.

$$\langle p \rangle_t = 0$$

: behaves like a matter.

$$\rho_a \approx \frac{1}{2} m_a A_1^2 \, m_1 \frac{R_1^3}{R^3}$$

$$m_1 = 3H(t_1) \sim \frac{T_1^2}{M_{\rm Pl}}$$ Oscillation temperature

$$\frac{R_1^3}{R^3} \sim \frac{T^3}{T_1^3}$$ Entropy conservation

$$\Rightarrow \quad m_1 \frac{R_1^3}{R^3} \sim \frac{T^3}{M_{\rm Pl}} \frac{1}{T_1} \sim \frac{T^3}{M_{\rm Pl}^{3/2}} \frac{1}{\sqrt{m_1}}$$

DM abundance

$$\rho_a \approx \frac{1}{2} m_a A_1^2 \, m_1 \frac{R_1^3}{R^3} \sim m_a A_1^2 \frac{1}{\sqrt{m_1}} \frac{T^3}{M_{\rm Pl}^{3/2}}$$

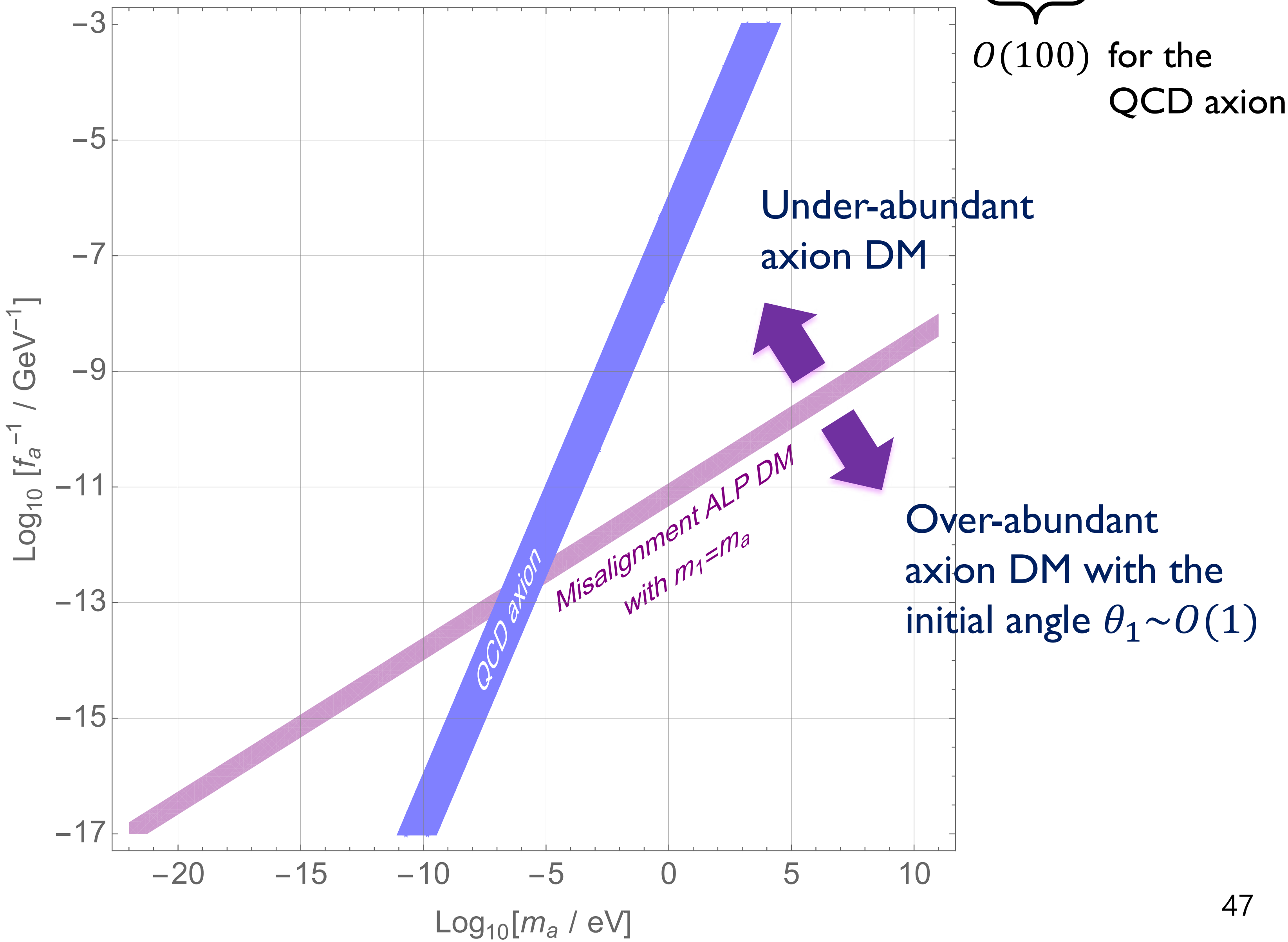
$$\Downarrow \quad \frac{A_1}{f_a} = \theta_1$$

$$\Omega_a h^2 \simeq 0.1 \sqrt{\frac{m_a}{1\,\mu\text{eV}}} \left(\frac{f_a}{10^{13}\,\text{GeV}} \right)^2 \theta_1^2 \left(\frac{g_{*S}(T_1)}{100} \right)^{-1/4} \times \underbrace{\sqrt{\frac{m_a}{m_1}}}_{O(100)}$$

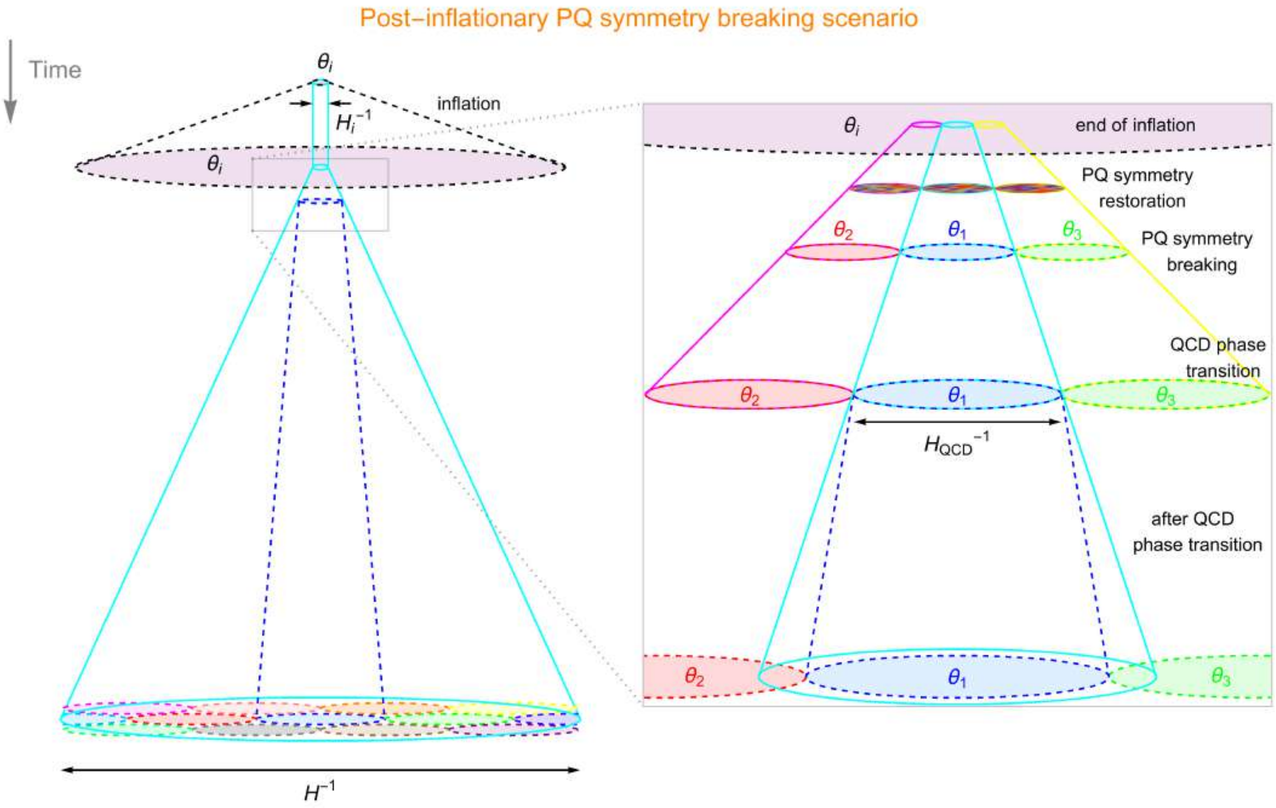
for the QCD axion

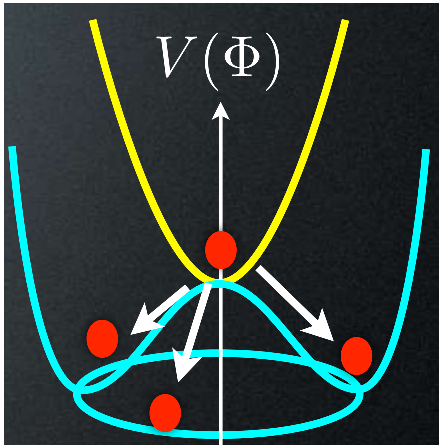
$$\Omega_a h^2 \simeq 0.1 \sqrt{\frac{m_a}{1\,\mu\mathrm{eV}}} \left(\frac{f_a}{10^{13}\,\mathrm{GeV}}\right)^2 \theta_1^2 \left(\frac{g_{*S}(T_1)}{100}\right)^{-1/4} \times \underbrace{\sqrt{\frac{m_a}{m_1}}}_{}$$

$O(100)$ for the QCD axion



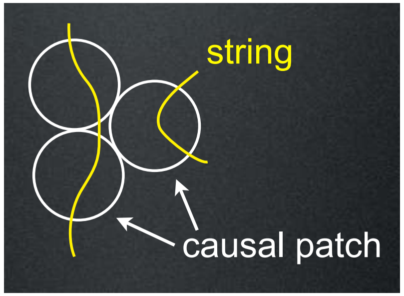
ii) PQ spontaneously broken after inflation





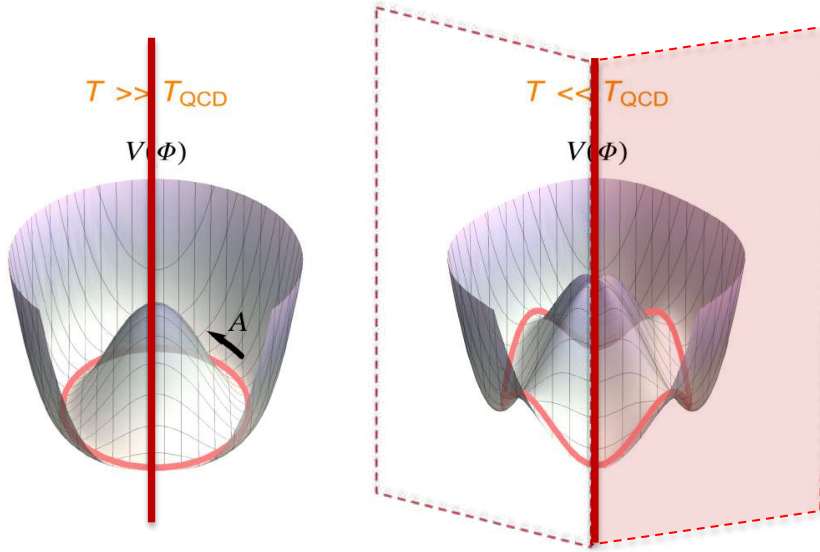
The axion field value can vary by O(1) over the horizon scale.

String-wall network in the post-inflationary PQ breaking scenario

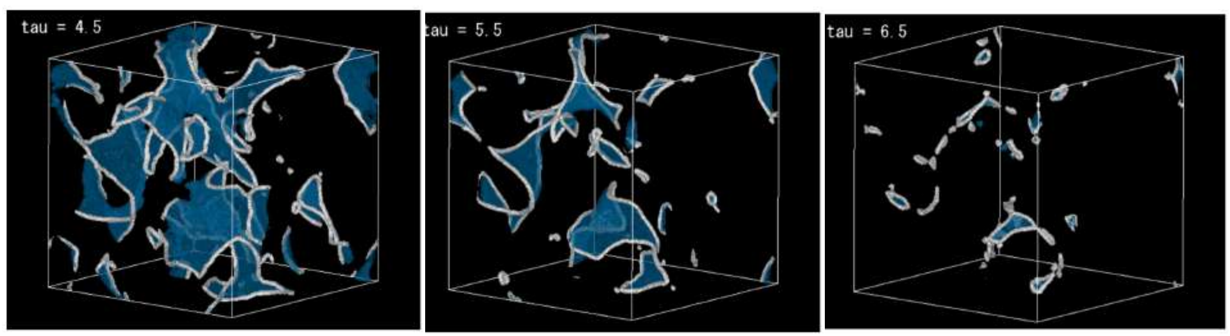


string

$$\mu_{\rm st} \sim f_a^2 \ln \left(\frac{f_a}{m_a} \right)$$



string +
domain walls



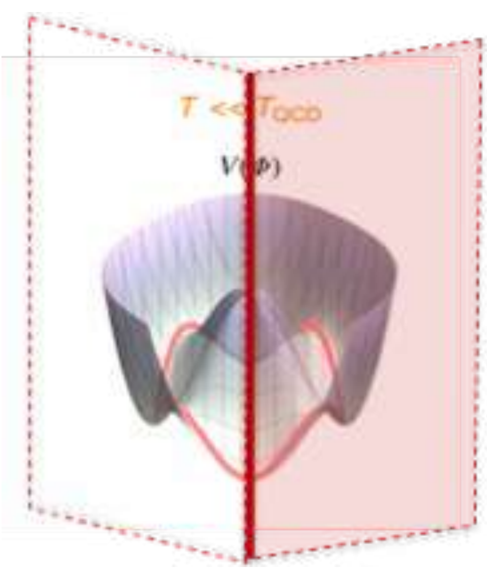
Hiramatsu, Kawasaki, Saikawa, Sekiguchi 1202.5851

The string-wall network decays to axions. The resultant axion production turns out to be larger than from the misalignment mechanism by about the log factor.

Axion Dark Matter

- PQ breaking *after* inflation

✓ Axionic strings and domain walls are generated.
✓ Axion DM is mainly given by annihilation of them.



string +
domain walls

But it requires $N_{DW} = 1$ for the string-wall network to collapse.

(Axion Domain Wall problem)

$$V(a) \simeq -m_u \Lambda_{\rm QCD}^3 \cos\left(N_{\rm DW} \frac{a(x)}{f_a} + \theta_{\rm QCD} \right)$$

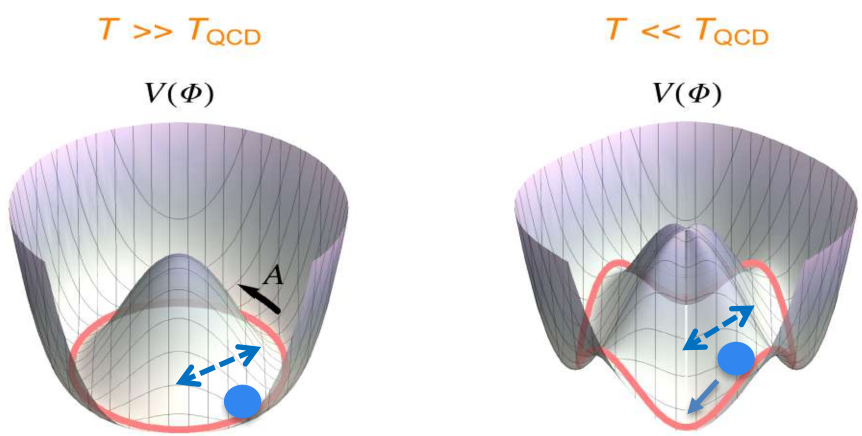
$$\frac{a(x)}{f_a} \equiv \frac{a(x)}{f_a} + 2\pi \quad \left(\text{i.e. } \frac{a(x)}{f_a} = \theta(x) \right)$$

(Note $N_{DW} = 6$ in the DFSZ model)

Axion Dark Matter

- PQ breaking *before/during* inflation

✓ No domain wall problem
✓ Axion DM is given by coherent oscillation of the axion field.

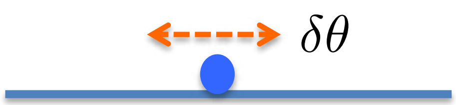


The axion field gets quantum fluctuation during inflation.

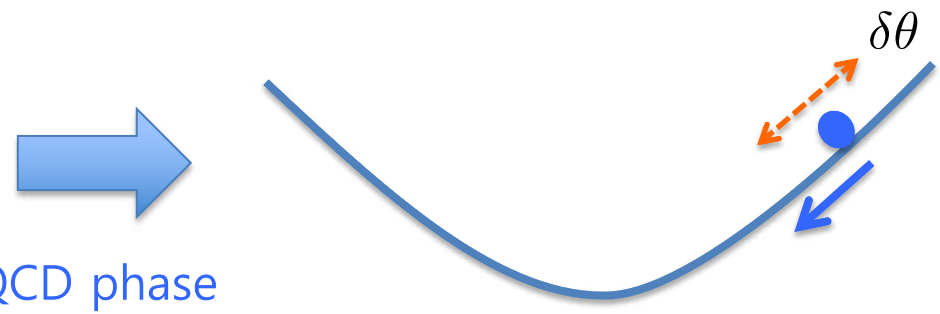
$$\delta a(x) = \frac{H_I}{2\pi}$$

Axionic isocurvature perturbation

- The primordial axion quantum fluctuations turn into axion DM density perturbation after QCD phase transition.



$$a(x) = f_a \theta(x)$$



QCD phase transition

$$\frac{\Omega_a}{\Omega_{\rm DM}} \simeq 1.7(\theta_0^2 + \delta\theta^2) \left(\frac{f_a(t_0)}{10^{12}\mathrm{GeV}} \right)^{1.19}$$

- It generates the "isocurvature mode" of CMB perturbation.

$$\left(\frac{\delta T}{T}\right)_{\rm iso} \simeq \frac{4}{5} \left(\frac{\Omega_a}{\Omega_{\rm DM}}\right) \frac{\delta\theta}{\theta_{\rm mis}} < 3.8 \times 10^{-6}$$

Planck Collaboration (2014)

Tension with *High* scale inflation

- If axion is a major component of DM in the Universe, the axion field fluctuations must be very small.

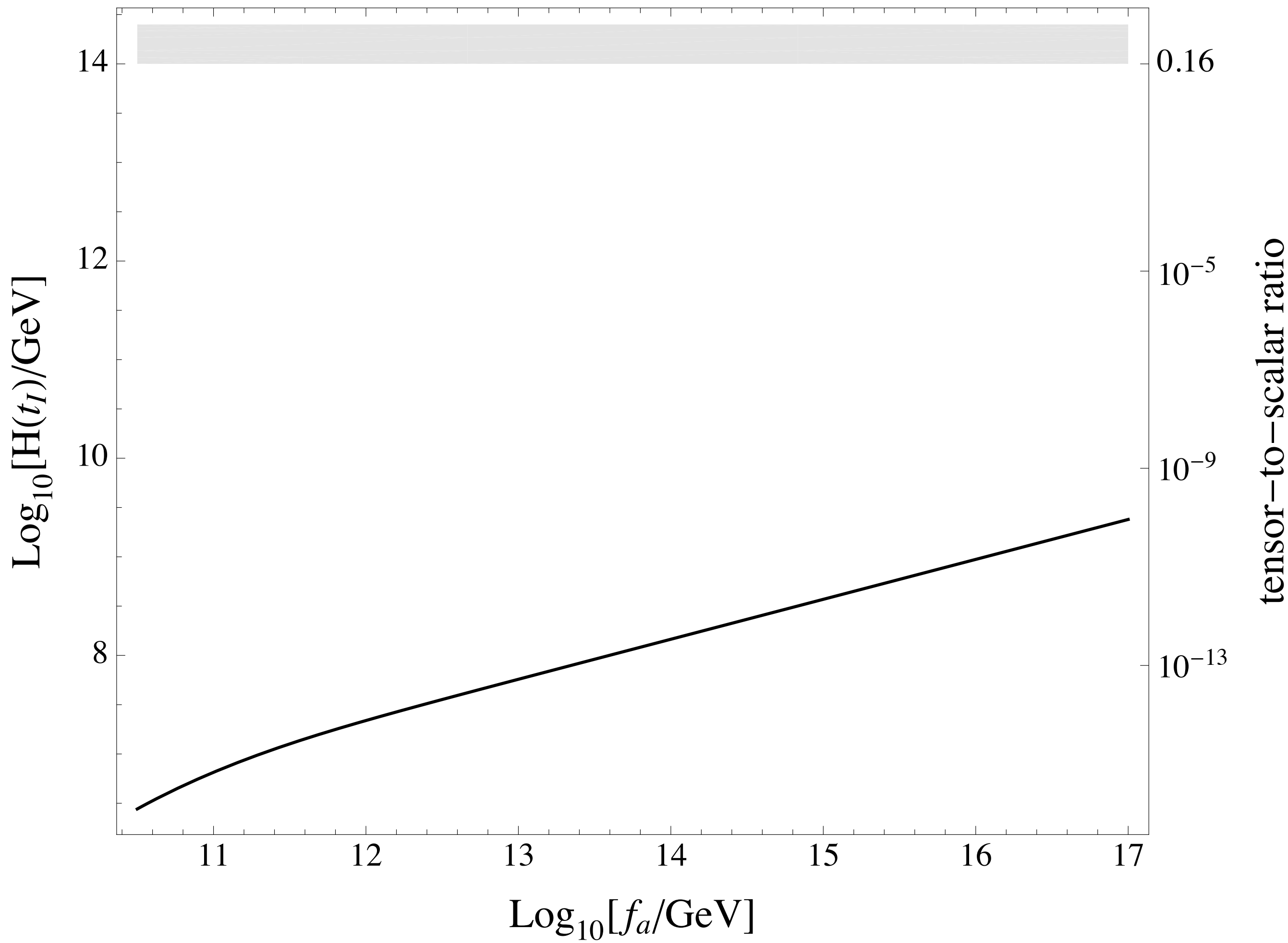
$$\left(\frac{\delta T}{T}\right)_{\rm iso} \simeq \frac{4}{5}\left(\frac{\Omega_a}{\Omega_{\rm DM}}\right)\frac{\delta\theta}{\theta_{\rm mis}} < 3.8\times 10^{-6} \quad \Longrightarrow \quad \frac{\delta\theta}{\theta_{\rm mis}} \lesssim 10^{-5}$$

$$\Omega_a \simeq \Omega_{\rm DM}$$

- This means that the primordial inflationary Hubble scale should be so suppressed compared to the axion scale at the inflationary epoch.

$$\delta\theta = \frac{H(t_I)}{2\pi f_a(t_I)} \quad \Longrightarrow \quad \boxed{H(t_I) \lesssim 10^{-5}\,\theta_{\rm mis}\, f_a(t_I)}$$

Upper bound on the inflation scale for $\Omega_a = \Omega_{DM}$ in the conventional scenario



Experimental targets for QCD axion-photon coupling

Axion effective interactions to the SM

$$\sum_{F=G,W,B} \frac{g_F^2}{32\pi^2} c_F \frac{a}{f_a} F^a_{\mu\nu} \widetilde{F}^{a\mu\nu} + \frac{\partial_\mu a}{f_a} \sum_{\psi=q,\ell} c_\psi \bar{\psi}\gamma^\mu\gamma^5\psi$$

At low energies below GeV

$$\frac{e^2}{32\pi^2} c_\gamma \frac{a}{f_a} F_{\mu\nu}\widetilde{F}^{\mu\nu} - \kappa_N \overbrace{\frac{a}{f_a}\frac{i}{2} F_{\mu\nu}(\bar{N}\sigma^{\mu\nu}\gamma^5 N)}^{\frac{a}{f_a}\vec{E}\cdot\vec{S}_N} + \frac{\partial_\mu a}{f_a}\left(c_N \bar{N}\gamma^\mu\gamma^5 N + c_e \bar{e}\gamma^\mu\gamma^5 e\right)$$

Photon coupling

$$c_\gamma = c_W + c_B - \frac{2}{3}\frac{(m_u+4m_d)}{(m_u+m_d)} c_G$$

Nucleon EDM coupling

$\kappa_n \simeq 2.4 \cdot 10^{-16}\, c_G$ e cm

(from QCD sum-rules)

Nucleon and electron derivative couplings

$$c_p \simeq -0.24\, c_G + 0.88\, c_u - 0.39\, c_d$$
$$c_n \simeq -0.01\, c_G - 0.39\, c_u + 0.88\, c_d$$

(from lattice QCD)

Basic detection principle on $g_{a\gamma}$

Axion-photon coupling

$$\mathcal{L} \supset \frac{g_{a\gamma}}{4} a F^{\mu\nu} \widetilde{F}_{\mu\nu} = -\frac{g_{a\gamma}}{4} a \vec{E} \cdot \vec{B}$$

modification of Maxwell's equations

$$\nabla \cdot \vec{E} = \rho + \overbrace{g_{a\gamma} \vec{B} \cdot \nabla a}^{\rho_{\text{eff}}}$$

$$\nabla \times \vec{B} = \partial_t \vec{E} + \vec{J} - \underbrace{g_{a\gamma} (\vec{B} \partial_t a + \vec{E} \times \nabla a)}_{\vec{J}_{\text{eff}}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\partial_t \vec{B}$$

$\partial_\mu a \neq 0$ → Effective electric charge and current → Axion-induced EM fields

Axion-photon coupling in KSVZ

$$y_Q \Phi Q_R^\dagger Q_L + h.c.$$

$$\Phi(x) = \frac{1}{\sqrt{2}}\left(f_a + \rho(x)\right) e^{i\frac{a(x)}{f_a}}$$

PQ symmetry : $\frac{a(x)}{f_a} \to \frac{a(x)}{f_a} + \alpha$ & $\begin{cases} Q_L(x) \to e^{i\,\xi_L \alpha} Q_L(x) \\ Q_R(x) \to e^{i\,\xi_R \alpha} Q_R(x) \end{cases}$ with $\xi_L - \xi_R = -1$

$\xi_{L(R)}$: PQ charge of $Q_{L(R)}$

Now redefine the quark fields as $\begin{cases} Q_L(x) \to e^{i\,\xi_L \frac{a(x)}{f_a}} Q_L(x) \\ Q_R(x) \to e^{i\,\xi_R \frac{a(x)}{f_a}} Q_R(x) \end{cases}$

Then the axion field in the Yukawa interaction disappears, but it appears again as …

$$Q_L(x) \to e^{i\,\xi_L \frac{a(x)}{f_a}} Q_L(x)$$

$$Q_R(x) \to e^{i\,\xi_R \frac{a(x)}{f_a}} Q_R(x)$$

with $\xi_L - \xi_R = -1$

$$Q_L^\dagger i\bar{\sigma}^\mu \partial_\mu Q_L \;\to\; Q_L^\dagger \bar{\sigma}^\mu \partial_\mu Q_L - \xi_L \frac{\partial_\mu a}{f_a} Q_L^\dagger\, \bar{\sigma}^\mu\, Q_L$$

$$Q_R^\dagger i\sigma^\mu \partial_\mu Q_R \;\to\; Q_R^\dagger \sigma^\mu \partial_\mu Q_R - \xi_R \frac{\partial_\mu a}{f_a} Q_R^\dagger\, \sigma^\mu Q_R$$

$$-\xi_L \frac{\partial_\mu a}{f_a} Q_L^\dagger\, \bar{\sigma}^\mu\, Q_L - \xi_R \frac{\partial_\mu a}{f_a} Q_R^\dagger\, \sigma^\mu Q_R$$

$$= (\xi_L - \xi_R) \frac{\partial_\mu a}{f_a} Q^\dagger \gamma^\mu \gamma^5 Q + (\xi_L + \xi_R) \frac{\partial_\mu a}{f_a} Q^\dagger \gamma^\mu Q \qquad Q = \begin{pmatrix} Q_L \\ Q_R \end{pmatrix}$$

$$= -\frac{\partial_\mu a}{f_a} Q^\dagger \gamma^\mu \gamma^5 Q + (2\xi_L + 1) \frac{\partial_\mu a}{f_a} Q^\dagger \gamma^\mu Q$$

Axial coupling (physical) — Vector coupling (unphysical: ξ_L is arbitrary.)

The derivative interactions to the quarks become negligible when $E < m_Q$

$$Q_L(x) \to e^{i\,\xi_L \frac{a(x)}{f_a}} Q_L(x)$$

$$Q_R(x) \to e^{i\,\xi_R \frac{a(x)}{f_a}} Q_R(x)$$

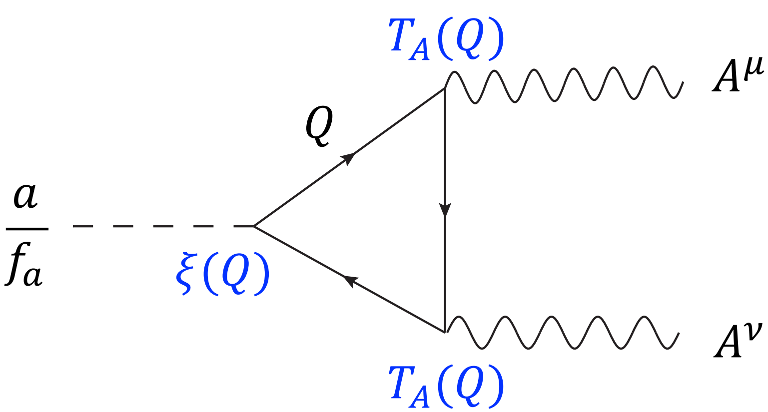
with $\xi_L - \xi_R = -1$

By the chiral anomaly,

$$\frac{a}{f_a} \sum_{A,Q} \xi(Q) \mathrm{Tr}[T_A^2(Q)] \frac{g_A^2}{16\pi^2} A^{\mu\nu} \tilde{A}_{\mu\nu}$$

$$= (-\xi_L + \xi_R) \frac{a}{f_a} \sum_A \mathrm{Tr}[T_A^2(Q)] \frac{g_A^2}{16\pi^2} A^{\mu\nu} \tilde{A}_{\mu\nu}$$

$$= \frac{a}{f_a} \sum_A \mathrm{Tr}[T_A^2(Q)] \frac{g_A^2}{16\pi^2} A^{\mu\nu} \tilde{A}_{\mu\nu}$$



E.g. in minimal KSVZ model, the quark's charge R_Q under the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is

$$R_Q = (3, 1, 0)$$

$Q_{L(R)}$ is (anti-)fundamental representation under $SU(3)_c$.

$$\frac{a}{f_a} \sum_A \mathrm{Tr}[T_A^2(Q)] \frac{g_A^2}{16\pi^2} A^{\mu\nu} \tilde{A}_{\mu\nu} = \frac{a}{f_a} \frac{g_3^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

: only axion-gluon coupling without axion-photon coupling at high energies above the QCD confinement scale ~ 1 GeV

$$\Rightarrow \quad -\frac{e^2}{32\pi^2} \frac{a}{f_a} \left(\frac{2}{3} \frac{4m_d + m_u}{m_u + m_d} \right) F^{\mu\nu} \tilde{F}_{\mu\nu} \simeq -\frac{e^2}{32\pi^2} \frac{a}{f_a} 1.92(4) F^{\mu\nu} \tilde{F}_{\mu\nu}$$

At low energies below the QCD scale ~ 1 GeV

: axion-photon coupling arising from the axion-gluon coupling due to axion-pion mixing

However, the minimal KSVZ model has a cosmological problem that the exotic quarks are stable and may be over-produced in the early universe beyond the observed dark matter density.

In order to allow the exotic quarks to decay, they have to be charged under the SM gauge group $U(1)_Y$.

An example: $R_Q = (3, 1, -1/3)$

➡ $Q_L^\dagger d_R$ allowed for $Q = \binom{Q_L}{Q_R}$ to decay

$$\frac{a}{f_a} \sum_A \mathrm{Tr}[T_A^2(Q)] \frac{g_A^2}{16\pi^2} A^{\mu\nu} \tilde{A}_{\mu\nu} = \frac{a}{f_a} \left(\frac{g_3^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu} + \frac{g_1^2}{16\pi^2} 3 \frac{1}{3^2} B^{\mu\nu} \tilde{B}_{\mu\nu} \right)$$

$$= \frac{a}{f_a} \left(\frac{g_3^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu} + \frac{e^2}{32\pi^2} \frac{2}{3} F^{\mu\nu} \tilde{F}_{\mu\nu} - 2 \frac{e^2 t_W}{16\pi^2} \frac{1}{3^2} F^{\mu\nu} \tilde{Z}_{\mu\nu} + \frac{e^2 t_W^2}{16\pi^2} \frac{1}{3^2} Z^{\mu\nu} \tilde{Z}_{\mu\nu} \right)$$

$B^{\mu\nu} = c_W F^{\mu\nu} - s_W Z^{\mu\nu}$ $\qquad g_1 = g_2 \tan\theta_W$

$e = g_2 \sin\theta_W = g_1 \cos\theta_W$

People commonly denote the anomaly coefficients as N and E which are defined as

$$\frac{a}{f_a}\left(\frac{g_3^2}{32\pi^2} N\, G^{\mu\nu}\, \tilde{G}_{\mu\nu} + \frac{e^2}{32\pi^2} E\, F^{\mu\nu}\, \tilde{F}_{\mu\nu}\right)$$

$$= \frac{a}{f_a{}'}\left(\frac{g_3^2}{32\pi^2}\, G^{\mu\nu}\, \tilde{G}_{\mu\nu} + \frac{e^2}{32\pi^2}\frac{E}{N}\, F^{\mu\nu}\, \tilde{F}_{\mu\nu}\right)$$

$$f_a' \equiv \frac{f_a}{N} \qquad \text{where} \qquad a(x) \equiv a(x) + 2\pi f_a$$

$$m_a \simeq N\frac{m_\pi f_\pi}{f_a} = \frac{m_\pi f_\pi}{f_a{}'} = 5.7\ \mu\text{eV}\left(\frac{10^{12}\ \text{GeV}}{f_a'}\right)$$

$$\frac{1}{4} g_{a\gamma} F^{\mu\nu}\, \tilde{F}_{\mu\nu}$$

$$g_{a\gamma} = \frac{e^2}{8\pi^2}\frac{1}{f_a{}'}\left(\frac{E}{N} - 1.92(4)\right)$$
$$\simeq \frac{1}{5\times10^{15}\ \text{GeV}}\left(\frac{m_a}{\mu\text{eV}}\right)\left(\frac{E}{N} - 1.92(4)\right)$$

$$\frac{E}{N} = 0\ [R_Q = (3,1,0)],$$
$$\frac{2}{3}\left[R_Q = \left(3,1,-\frac{1}{3}\right)\right],$$
$$\ldots$$

L D Luzio, F Mescia, E Nardi '16 redefined the "QCD axion window" by two requirements

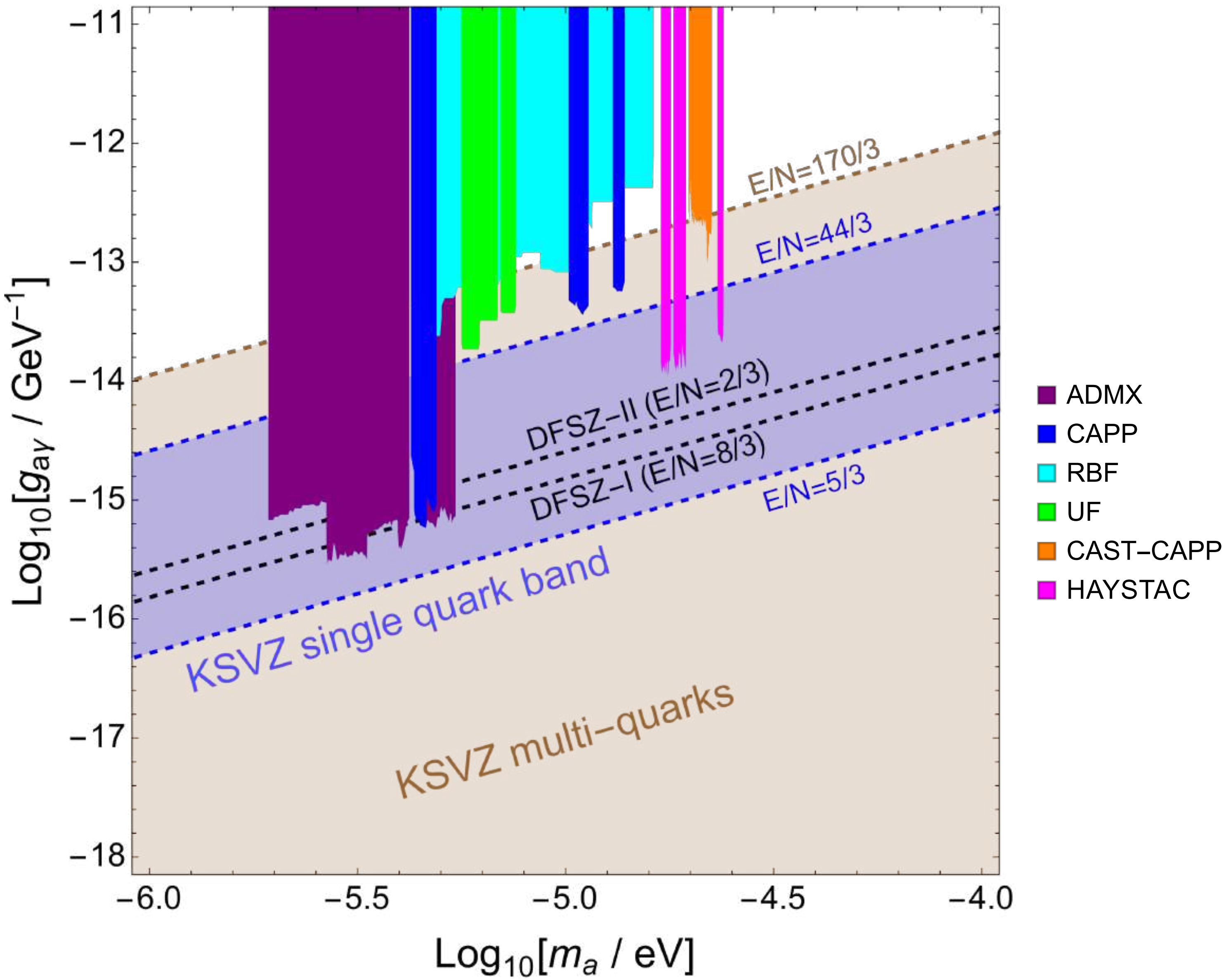
i) Cosmologically unstable exotic quarks (Not to overclose the universe, constraints from stellar dynamics, BBN constraints) → The exotic quarks must carry non-zero $U(1)_Y$ charge.

ii) No Landau pole of the gauge couplings up to the Planck scale → The representation (or charges) of an exotic quark under the SM gauge groups should be not too big.

R_Q	$\mathcal{O}_{Qq}$	$\Lambda_{LP}^{R_Q}$[GeV]	E/N	N_{DW}
$R_1: (3, 1, -\frac{1}{3})$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38} (g_1)$	2/3	1
$R_2: (3, 1, +\frac{2}{3})$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34} (g_1)$	8/3	1
$R_3: (3, 2, +\frac{1}{6})$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39} (g_1)$	5/3	2
$R_4: (3, 2, -\frac{5}{6})$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27} (g_1)$	17/3	2
$R_5: (3, 2, +\frac{7}{6})$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22} (g_1)$	29/3	2
$R_6: (3, 3, -\frac{1}{3})$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30} (g_2)$	14/3	3
$R_7: (3, 3, +\frac{2}{3})$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27} (g_2)$	20/3	3
$R_8: (3, 3, -\frac{4}{3})$	$\overline{Q}_L d_R H^{\dagger 2}$	$3.5 \cdot 10^{18} (g_1)$	44/3	3
$R_9: (\overline{6}, 1, -\frac{1}{3})$	$\overline{Q}_L \sigma d_R \cdot G$	$2.3 \cdot 10^{37} (g_1)$	4/15	5
$R_{10}: (\overline{6}, 1, +\frac{2}{3})$	$\overline{Q}_L \sigma u_R \cdot G$	$5.1 \cdot 10^{30} (g_1)$	16/15	5
$R_{11}: (\overline{6}, 2, +\frac{1}{6})$	$\overline{Q}_R \sigma q_L \cdot G$	$7.3 \cdot 10^{38} (g_1)$	2/3	10
$R_{12}: (8, 1, -1)$	$\overline{Q}_L \sigma e_R \cdot G$	$7.6 \cdot 10^{22} (g_1)$	8/3	6
$R_{13}: (8, 2, -\frac{1}{2})$	$\overline{Q}_R \sigma \ell_L \cdot G$	$6.7 \cdot 10^{27} (g_1)$	4/3	12
$R_{14}: (15, 1, -\frac{1}{3})$	$\overline{Q}_L \sigma d_R \cdot G$	$8.3 \cdot 10^{21} (g_3)$	1/6	20
$R_{15}: (15, 1, +\frac{2}{3})$	$\overline{Q}_L \sigma u_R \cdot G$	$7.6 \cdot 10^{21} (g_3)$	2/3	20

$$0.25 \leq \left|\frac{E}{N} - 1.92(4)\right| \leq 12.75$$

$$\left|\frac{E}{N} - 1.92(4)\right| = 0.75 \text{ (DFSZ I)}, 1.25 \text{ (DFSZ II)}$$



−11
−12
−13
−14
−15
−16
−17
−18
−6.0
−5.5
−5.0
−4.5
−4.0
$\mathrm{Log}_{10}[g_{a\gamma}$ / GeV^{-1}]
$\mathrm{Log}_{10}[m_a$ / eV]
E/N=170/3
E/N=44/3
DFSZ-II (E/N=2/3)
DFSZ-I (E/N=8/3)
E/N=5/3
KSVZ single quark band
KSVZ multi−quarks
ADMX
CAPP
RBF
UF
CAST−CAPP
HAYSTAC