Cosmic Inflation

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The 1st Gülbahçe School and Workshop for Theoretical Physics

References

Quantum Field Theory and General Relativity

“The early Universe”, Kolb and Turner

“Physical foundations of cosmology”, Mukhanov

“Cosmology”, Weinberg

“Particle physics and inflationary cosmology”, Andrei Linde, hep-th/0503203

“TASI lectures on inflation”, Daniel Baumann, arXiv:0907.5424

“What do we learn from the CMB observations?”, Rubakov and Vlasov, 1008.1704

and more

More in the arXiv: http://xxx.lanl.gov

in SpiresHep: https://inspirehep.net/?ln=en

Cosmology

To understand the origin, evolution and future of the Universe as a whole.

Cosmic expansion and Big Bang cosmology

The Universe is expanding from the early hot and dense state.
The standard model of cosmology.

Cosmic Inflation

Inflation can explain why the Universe is homogeneous and isotropic.

Inflation also explain the small perturbation that generates the CMB anisotropy and ultimately seed the structure formation of large scales.

Cosmic expansion and Big Bang cosmology

• Unit of distance

1 AU (Astronomical Unit)

: Average distance between Sun and Earth

$= 1.49 \times 10^8$ km (8 min 18 sec with light)

1 Ly (Light year): the light travels for 1 year

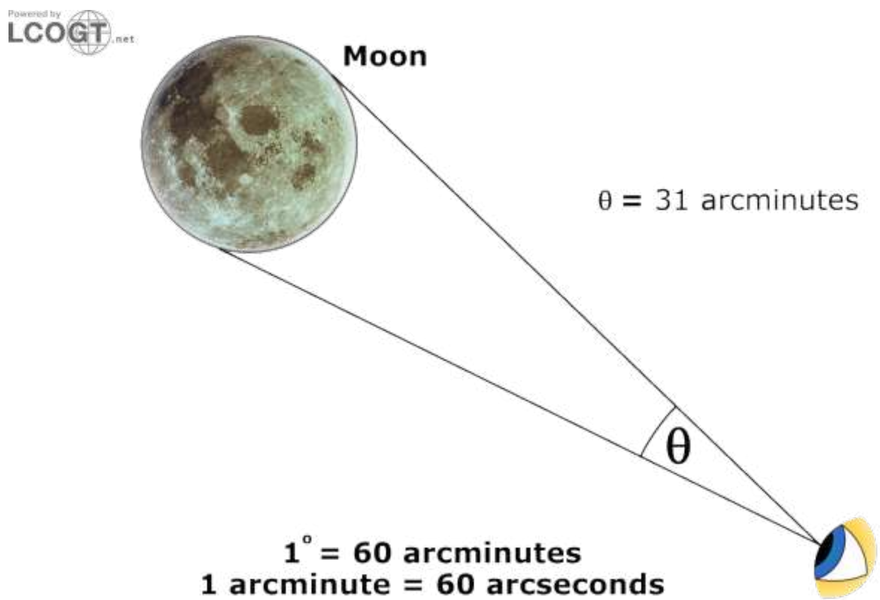
speed of light x 1 year = 9.46×10^{12} km

1 pc (parsec) ~ 3.26 Ly

1 Arcmin = 1 degree / 60

1 Arcsec = 1 Arcmin / 60

=1 degree / 3600



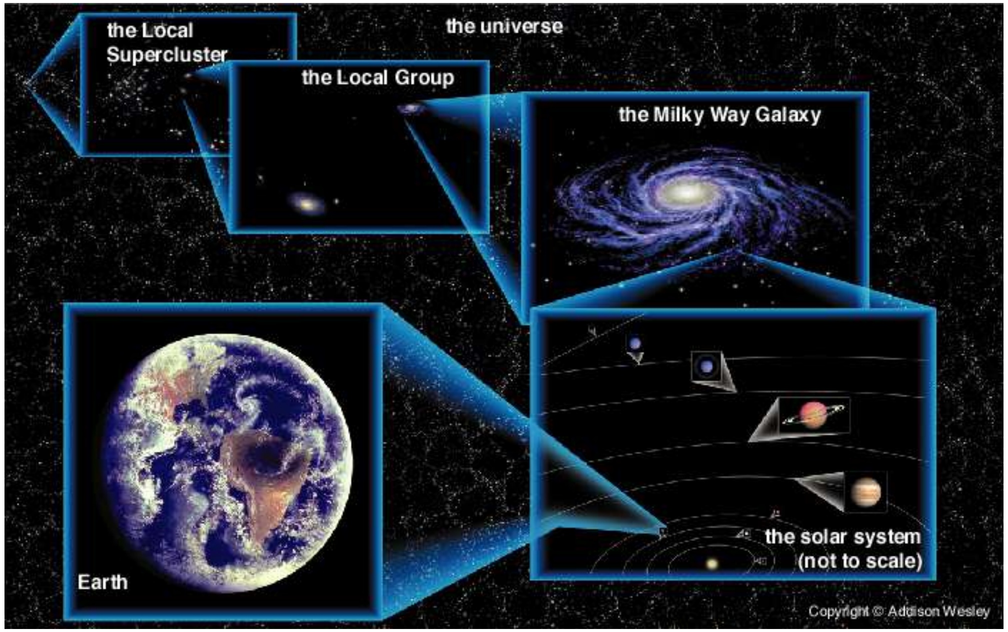
Between stars in the galaxy ~ 10 pc

Size of the galaxy ~ 10 kpc

Between galaxies ~ Mpc

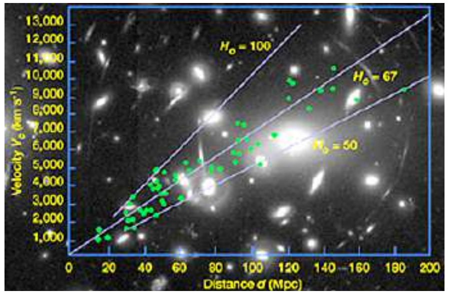
Size of cluster of galaxies ~ 10 Mpc

The observable Universe ~ 30 Gpc

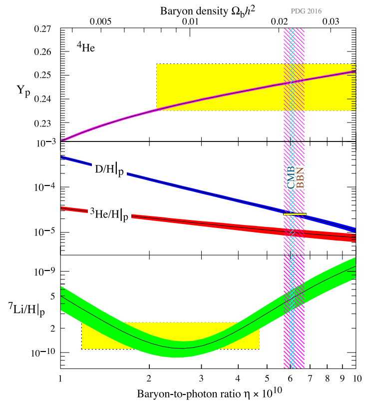


Standard Big Bang cosmology explains

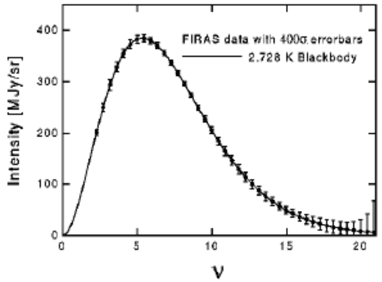
1. the expanding Universe
   : Hubble's law



2. the abundance of the light elements
   : Big Bang Nucleosynthesis

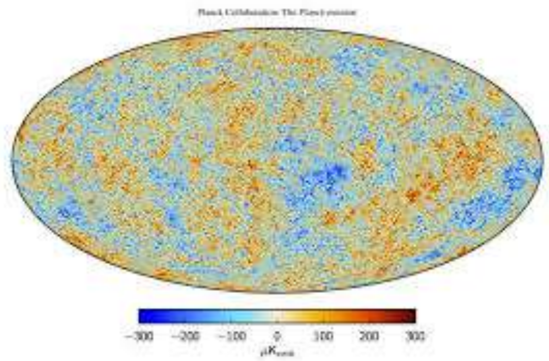


3. the cosmic microwave background radiation
   : Black body CMB

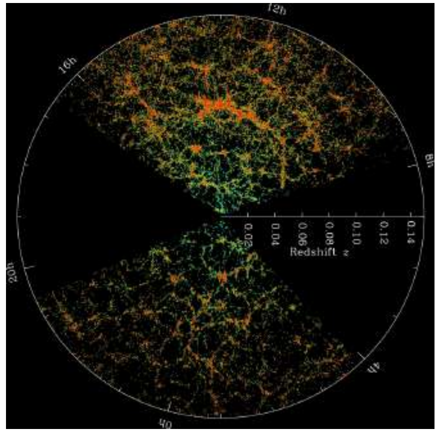


* Assuming that the Universe is homogeneous, isotropic and flat initially, which can be explained by cosmic inflation.

with density perturbations



1. temperature anisotropy of CMB
   : power spectrum



2. structure formation of large scales
   : galaxy, cluster of galaxies

* The cosmic inflation can explain the generation of the initial density perturbations.

• Standard Big Bang model: general relativity

Big bang cosmology assumes that the matter and radiation are uniformly distributed throughout the Universe - Cosmological principle
It is true for large scales of larger than around 100 Mpc for matter distribution and also in the temperature distribution observed in the Cosmic Microwave Background radiation.

The metric for a homogeneous and isotropic space is Friedmann-Robertson-Walker (FRW) metric with line element

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin\theta^2 d\phi^2\right\}$$

R(t) : scale factor

$k > 0$ closed, $k = 0$ flat, $k < 0$ open

* The assumption of homogeneity and isotropy is understood locally and global properties can be different.

• Standard Big Bang model: General Relativity

Einstein equation

$$\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

matter in the Universe

perfect fluid

energy-momentum

geometry : metric $g^{\mu\nu}$

$T^{\mu}_{\nu} = \mathrm{diag}(\rho, -p, -p, -p)$

The metric for a homogeneous and isotropic space is Friedmann-Robertson-Walker (FRW) metric with line element

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin\theta^2 d\phi^2\right\}$$

1. Evolving Universe: scale factor R(t), redshift z

$$\boxed{\frac{\lambda_1}{\lambda_0} = \frac{R(t_1)}{R(t_0)} \equiv 1 + z}$$

2. Momentum redshift $\boxed{P(t) \propto 1/a(t)}$

In General Relativity, the dynamical variables characterizing the gravitational field are the components of the metric $g_{\alpha\beta}(x^\gamma)$ and they obey the Einstein equations:

$$G^\alpha_\beta \equiv R^\alpha_\beta - \frac{1}{2}\delta^\alpha_\beta R - \Lambda\delta^\alpha_\beta = 8\pi G T^\alpha_\beta. \tag{1.48}$$

Here

$$R^\alpha_\beta = g^{\alpha\gamma}\left(\frac{\partial\Gamma^\delta_{\gamma\beta}}{\partial x^\delta} - \frac{\partial\Gamma^\delta_{\gamma\delta}}{\partial x^\beta} + \Gamma^\delta_{\gamma\beta}\Gamma^\sigma_{\delta\sigma} - \Gamma^\sigma_{\gamma\delta}\Gamma^\delta_{\beta\sigma}\right) \tag{1.49}$$

is the Ricci tensor expressed in terms of the inverse metric $g^{\alpha\gamma}$, defined via $g^{\alpha\gamma}g_{\gamma\beta} = \delta^\alpha_\beta$, and the Christoffel symbols

$$\Gamma^\alpha_{\gamma\beta} = \frac{1}{2}g^{\alpha\delta}\left(\frac{\partial g_{\gamma\delta}}{\partial x^\beta} + \frac{\partial g_{\delta\beta}}{\partial x^\gamma} - \frac{\partial g_{\gamma\beta}}{\partial x^\delta}\right). \tag{1.50}$$

The symbol δ^α_β denotes the unit tensor, equal to 1 when $\alpha = \beta$ and 0 otherwise; $R = R^\alpha_\alpha$ is the scalar curvature; and $\Lambda = $ const is the cosmological term. Matter

[Mukhanov, Physical Foundations of cosmology]

Redshift: z

The wavelength of light is redshifted due to the expansion $\lambda = \frac{h}{p}$

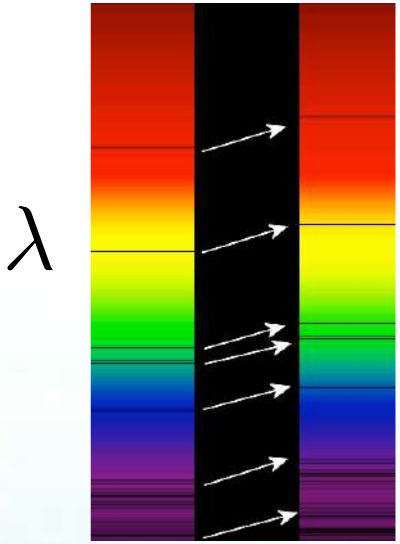
$$\frac{\lambda_1}{\lambda_0} = \frac{R(t_1)}{R(t_0)} \equiv 1 + z$$

Therefore the redshift is said to have radial velocity cz,
so the existence of distant sources with z > 1 does not imply any violation of the special relativity.

The redshift depends on the increase of scale factor in the whole period from emission to absorption.

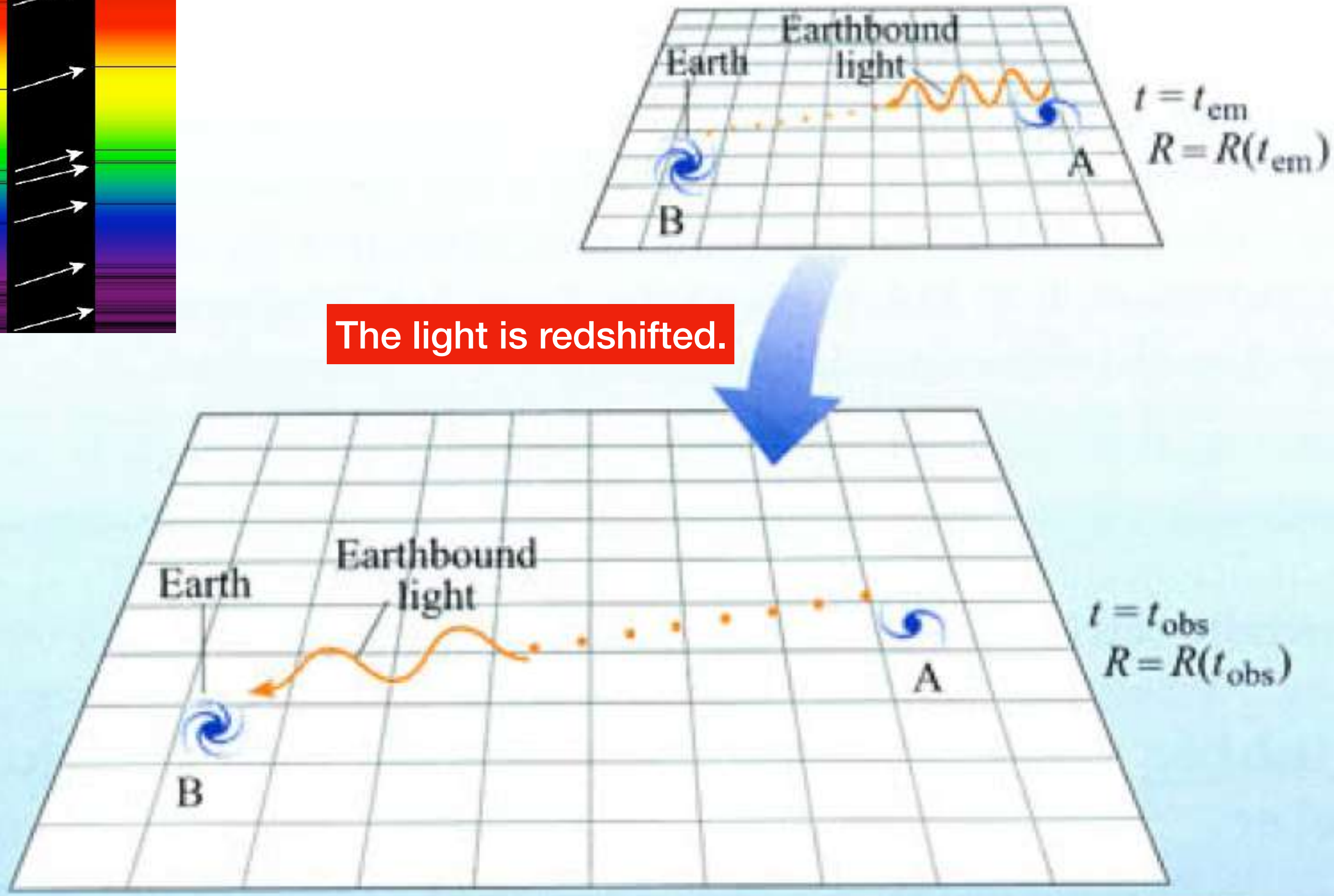
Since the peculiar velocity is hundred km/s, to see the redshift due to expansion, z >> 0.001 c.

Redshift



Hubble-Lemaitre Law

*2018 IAU(International Astronomical Unit)



Redshift of momentum

The motion of a particle that is not at rest in the comoving coordinate system.

The momentum in GR $P \equiv m_0 \sqrt{g_{ij} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau}}$ with $d\tau^2 = dt^2 - h_{ij} dx^i dx^j$

From eq. of motion, considering the particle position near the origin,

$$\frac{d^2 x^i}{d\tau^2} = -\Gamma^i_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -\frac{2}{a} \frac{da}{dt} \frac{dx^i}{d\tau} \frac{dt}{d\tau} .$$

ignoring spatial components Γ^i_{jk} of the affine connection.

Multiplying with $d\tau/dt$ gives

$$\frac{d}{dt}\left(\frac{dx^i}{d\tau}\right) = -\frac{2}{a} \frac{da}{dt} \frac{dx^i}{d\tau} , \longrightarrow \frac{dx^i}{d\tau} \propto \frac{1}{a^2(t)} .$$

Therefore

with metric $h_{ij} = a^2(t) \delta_{ij}$

$$\boxed{P(t) \propto 1/a(t)}$$

Friedmann-Robertson-Walker metric in polar coordinates

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]$$

(t, r, θ, φ) comoving coordinates

$R(t)$ scale factor

$k = +1, 0, -1$ spatial curvature

$$g_{00} = 1,\ g_{rr} = -\frac{R^2(t)}{1-kr^2},$$

$$g_{\theta\theta} = R^2(t)r^2,\ g_{\phi\phi} = R^2(t)r^2\sin^2\theta$$

Circumference of coordinate r and φ

$$\int R(t) r d\theta = 2\pi R(t) r$$

Area of coordinate r

$$\int R^2(t) r^2 d\theta d\varphi = 4\pi R^2(t) r^2$$

The equation of motion of freely falling particles

$$\boxed{\frac{d^2x^\mu}{du^2} + \Gamma^\mu_{\nu\kappa}\frac{dx^\nu}{du}\frac{dx^\kappa}{du} = 0}$$

with affine connection $\Gamma^\mu_{\nu\kappa} = \frac{1}{2}g^{\mu\lambda}\left[\frac{\partial g_{\lambda\nu}}{\partial x^\kappa} + \frac{\partial g_{\lambda\kappa}}{\partial x^\nu} - \frac{\partial g_{\nu\kappa}}{\partial x^\lambda}\right]$

1. Initially at rest $\frac{dx^i}{du} = 0$

then $\frac{d^2x^i}{du^2} = -\Gamma^i_{jk}\frac{dx^j}{du}\frac{dx^k}{du} = 0$ using $\Gamma^i_{00} = 0$

so always at rest in these coordinates,
so called comoving coordinates.

2. For this observer, the proper time is just dt,

so t is the time measured by the comoving observer.

Proper (physical) distance from origin to the comoving object along the radial direction at time t, $dt = d\theta = d\phi = 0$

$$d(r,t) = \int ds = R(t)\int_0^r \frac{dr}{\sqrt{1-kr^2}} = R(t) \times \begin{cases} \sin^{-1} r & k = +1 \\ r & k = 0 \\ \sinh^{-1} r & k = -1 \end{cases}$$

The proper distance between two co-moving objects is proportional to the scale factor R(t). The rate of change of the proper distance is just

$$\dot{d} = d\frac{\dot{R}}{R} \qquad H \equiv \frac{\dot{R}(t)}{R(t)}$$

Hubble parameter

Particle horizon : The boundary that the information transfer is limited by the finite age of Universe and speed of light. It can be finite or infinite.

geodesic of light

$$ds^2 = 0 \longrightarrow \int_0^t \frac{dt'}{R(t')} = \int_0^{r_p} \frac{dr}{\sqrt{1-kr^2}}$$

Comoving distance r_p : light have traveled since t=0

Physical distance of particle horizon d_p

$$d_p(t) = R(t)\int_0^t \frac{dt'}{R(t')} = R(t)\int_0^{r_p} \frac{dr}{\sqrt{1-kr^2}}$$

Conformal time $\quad d\eta = \frac{dt}{a(t)} \qquad = R(t)[\eta(t) - \eta(t=0)]$

$$ds^2 = R^2(t)\left[d\eta^2 - \left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right\}\right]$$

Friedmann-Robertson-Walker metric in 4-dim space-time

$$ds^2 = dt^2 - R(t)^2 \left[d\vec{x}^2 + k \frac{(\vec{x} \cdot d\vec{x})^2}{1 - k\vec{x}^2} \right]$$

The components of the metric

$$g_{00} = 1 \qquad g_{0i} = 0 \qquad g_{ij} = -R^2(t) \left(\delta_{ij} + k \frac{x^i x^j}{1 - k\vec{x}^2} \right)$$

Affine connection

$$\Gamma^0_{ij} = \frac{\dot{R}}{R} h_{ij}$$

$$\Gamma^i_{oj} = \frac{\dot{R}}{R} \delta^i_j$$

$$\Gamma^i_{jk} = \frac{1}{2} h^{il} (h_{lj,k} + h_{lk,j} - h_{jk,l})$$

with $h_{ij} = -g_{ij}$

Ricci tensor

$$R_{00} = -3 \frac{\ddot{R}}{R}$$

$$R_{ij} = -\left[\frac{\ddot{R}}{R} + 2 \frac{\dot{R}^2}{R^2} + \frac{2k}{R^2} \right] g_{ij}$$

Ricci scalar

$$\mathcal{R} = -6 \left[\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} \right]$$

In General Relativity, the dynamical variables characterizing the gravitational field are the components of the metric $g_{\alpha\beta}(x^\gamma)$ and they obey the Einstein equations:

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The symbol δ^α_β denotes the unit tensor, equal to 1 when $\alpha = \beta$ and 0 otherwise; $R = R^\alpha_\alpha$ is the scalar curvature; and Λ = const is the cosmological term. Matter

[Mukhanov, Physical Foundations of cosmology] gives (0,0) component in the zero-th order : Friedmann equation

$$H \equiv \frac{\dot{R}(t)}{R(t)}$$

$$\boxed{H^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho} \quad \textbf{(Eq.2)}$$

gives (i,i) component in the zero-th order

$$\boxed{2\frac{\ddot{R}}{R} + H^2 + \frac{k}{R^2} = -8\pi G p} \quad \textbf{(Eq.3)}$$

The difference gives

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p)$$

for k=1

$$q_0 \equiv -\frac{1}{H_0^2}\frac{\ddot{R}(t_0)}{R(t_0)} = \Omega_0 \frac{1+3\omega}{2}$$

decelelation parameter

Perfect fluid

: completely characterized by its rest frame energy density and isotropic pressure

$$T_{00} = \rho, \quad T_{0i} = T_{i0} = 0, \quad T_{ij} = \delta_{ij} p$$

In a general gravitational field and in the frame with velocity,

$$T_{\mu\nu} = (p+\rho) u_\mu u_\nu - p g_{\mu\nu} \qquad \text{with} \;\; g_{\mu\nu} u^\mu u^\nu = +1$$

The energy momentum tensor should be this form in the zeroth order due to the translational and rotational invariance.

Energy momentum conservation

$$\frac{\partial T^{\alpha\beta}}{\partial x^\beta} = 0\,. \quad \longrightarrow \quad T^{\mu\nu}{}_{;\nu} \equiv \frac{\partial T^{\mu\nu}}{\partial x^\nu} + \Gamma^\mu_{\kappa\nu} T^{\kappa\nu} + \Gamma^\nu_{\kappa\nu} T^{\mu\kappa} = 0$$

w/o gravity

in the presence of gravitational field

$$
\begin{aligned}
0 = \nabla_\mu T^{0\mu} =& \frac{\partial T^{0\mu}}{\partial x^\mu} + \Gamma^0_{\mu\nu} T^{\nu\mu} + \Gamma^\mu_{\mu\nu} T^{0\nu} \\
=& \frac{d\rho}{dt} + \Gamma^0_{ij} T^{ij} + \Gamma^i_{i0} T^{00} \\
=& \frac{d\rho}{dt} - \frac{\dot{a}}{a} g_{ij} T^{ij} + 3\frac{\dot{a}}{a} T^{00} \\
=& \frac{d\rho}{dt} + 3\frac{\dot{a}}{a} p + 3\frac{\dot{a}}{a} \rho
\end{aligned}
$$

Non-vanishing components of affine connections

$$
\begin{aligned}
\Gamma^0_{ij} =& \frac{\dot{R}}{R} h_{ij} \\
\Gamma^i_{oj} =& \frac{\dot{R}}{R} \delta^i_j \\
\Gamma^i_{jk} =& \frac{1}{2} h^{il} (h_{lj,k} + h_{lk,j} - h_{jk,l})
\end{aligned}
$$

$$
h_{ij} = -g_{ij}
$$

Energy-momentum conservation:

Continuity equation

$$T^{\mu\nu}{}_{;\nu} = 0 \xrightarrow[\mu = 0]{} \boxed{\dot{\rho} + 3H(\rho + p) = 0} \quad \textbf{(Eq.1)}$$

or $\quad d(\rho R^3) = -pd(R^3)$

with equation of state

$$\boxed{p = p(\rho)}$$

Radiation $\quad (p = \frac{1}{3}\rho)$

(Cold) Matter $\quad (p = 0)$

Vacuum energy
(cosmological constant) $\quad (p = -\rho)$

Eqs. 1, 2, 3 are connected by Bianchi identities and only two are independent.
Usually Eqs. 1 and 2 are taken.

Many kinds of matter content of our interest can be described by a simple eq. of state

$$\boxed{p = \omega\rho}$$

We can solve the continuity equation, $\boxed{\dot{\rho} + 3H(\rho + p) = 0}$

$$\frac{d\rho}{dt} + 3\frac{\dot{R}}{R}(1+\omega)\rho = 0$$

$$\frac{d\rho}{\rho} = -3(1+\omega)\frac{dR}{R}$$

$$\log\frac{\rho}{\rho_0} = -3(1+\omega)\log\frac{R}{R_0}$$

$$\rho(t) = \rho(t_0)\left(\frac{R(t)}{R(t_0)}\right)^{-3(1+\omega)}$$

Many kinds of matter content of our interest can be described by a simple eq. of state

$$\boxed{p = \omega\rho}$$

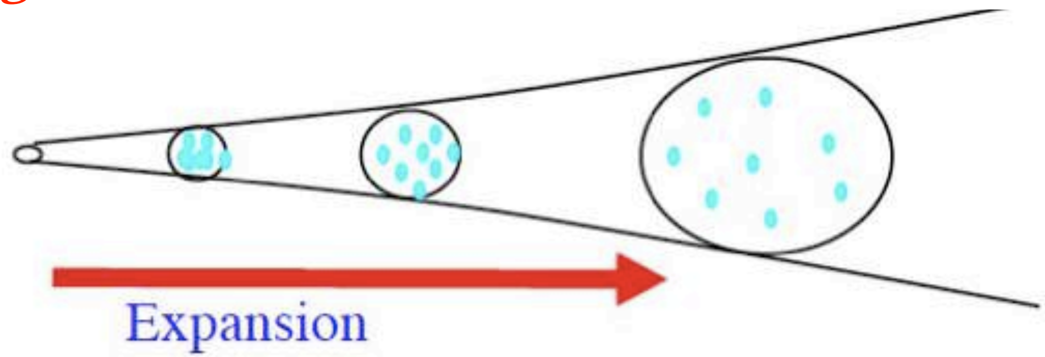
For constant w, we can solve and obtain

$$\rho \propto R^{-3(1+\omega)}$$

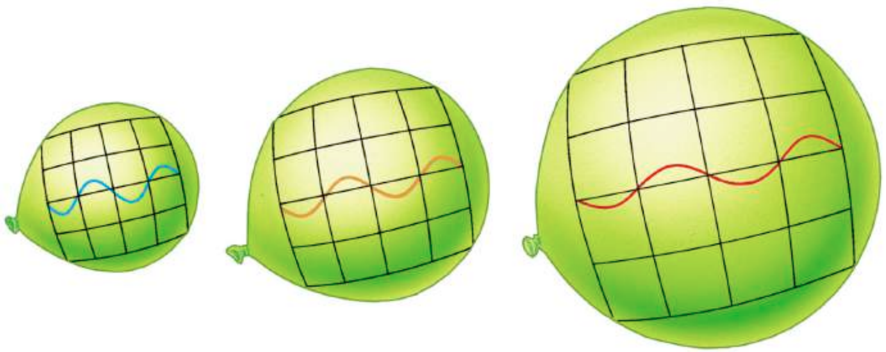
$$\left(\begin{array}{lll} \rho \propto R^{-4} & \text{Radiation} & (p = \frac{1}{3}\rho) \\ \rho \propto R^{-3} & \text{(Cold) Matter} & (p = 0) \\ \rho \propto \text{const} & \text{Vacuum energy (cosmological constant)} & (p = -\rho) \end{array} \right.$$

These apply separately for coexsting radiation, matter and cosmological constant, if there is no interactions between them.

In the expanding Universe



1. The number density decreases. $n(t) \propto \dfrac{1}{R^3(t)}$
2. The momentum is redshifted. $P(t) \propto \dfrac{1}{R(t)}$



Relativistic matter $\rho_r \sim E(t)\, n(t) \propto R(t)^{-4}$

→ Non-Relativistic matter $\rho_m \sim M\, n(t) \propto R(t)^{-3}$

Scale factor as a function of time

We can solve Friedmann equation with a given matter type

For k=0 (flat), and $p = \omega\rho$ then from $H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho_i\left(\frac{R}{R_i}\right)^{-3(1+\omega)}$

we obtain $\boxed{R \propto t^{2/3(1+\omega)}}$

Radiation $(p = \frac{1}{3}\rho)$ $R \propto t^{1/2}$

Matter $(p = 0)$ $R \propto t^{2/3}$

Vacuum energy $(p = -\rho)$ $R \propto \exp(Ht)$

The age of the Universe

$$\frac{R_0}{R} = 1 + z \equiv x^{-1}$$

$$t = \int_0^t dt' = \int_0^{R(t)} \frac{dR'}{\dot{R}'}$$

$$= H_0^{-1} \int_0^{(1+z)^{-1}} \frac{dx}{\left[1 - \Omega_0 + \sum_i \Omega_{i0} x^{-1-3\omega_i}\right]^{1/2}}$$

For flat Universe, $(\Omega_0 = 1)$, the age of the present universe is

$$t_0 = \frac{2}{3(1+\omega)} H_0^{-1}$$

$$t_0 = \frac{1}{2} H_0^{-1} \qquad \text{Radiation} \quad \left(p = \frac{1}{3}\rho\right)$$

$$t_0 = \frac{2}{3} H_0^{-1} \qquad \text{Matter} \ (p = 0)$$

$$H_0^{-1} = 9.78 h^{-1} \times 10^9 \text{ yr}$$
$$= 3000 h^{-1} \text{ Mpc}$$

$$\frac{2}{3} H_0^{-1} \simeq 9.3 \ Gyrs$$

Critical density

Friedmann equation

$$\boxed{H^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho}$$

$$\frac{k}{R^2 H^2} = \frac{\rho}{3H^2/8\pi G} - 1 \equiv \Omega - 1$$

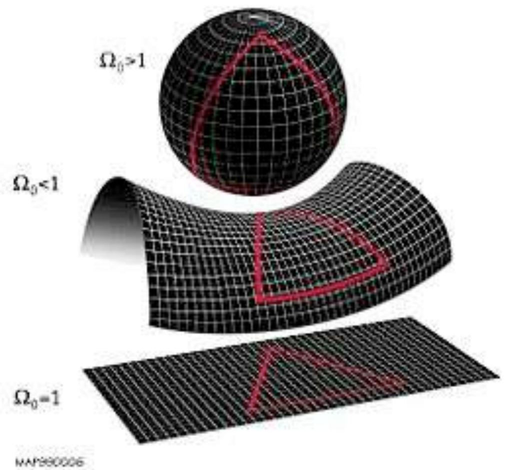
Critical density $$\rho_c \equiv \frac{3H^2}{8\pi G}$$

Density parameters $$\Omega_i \equiv \frac{\rho_i}{\rho_c}$$

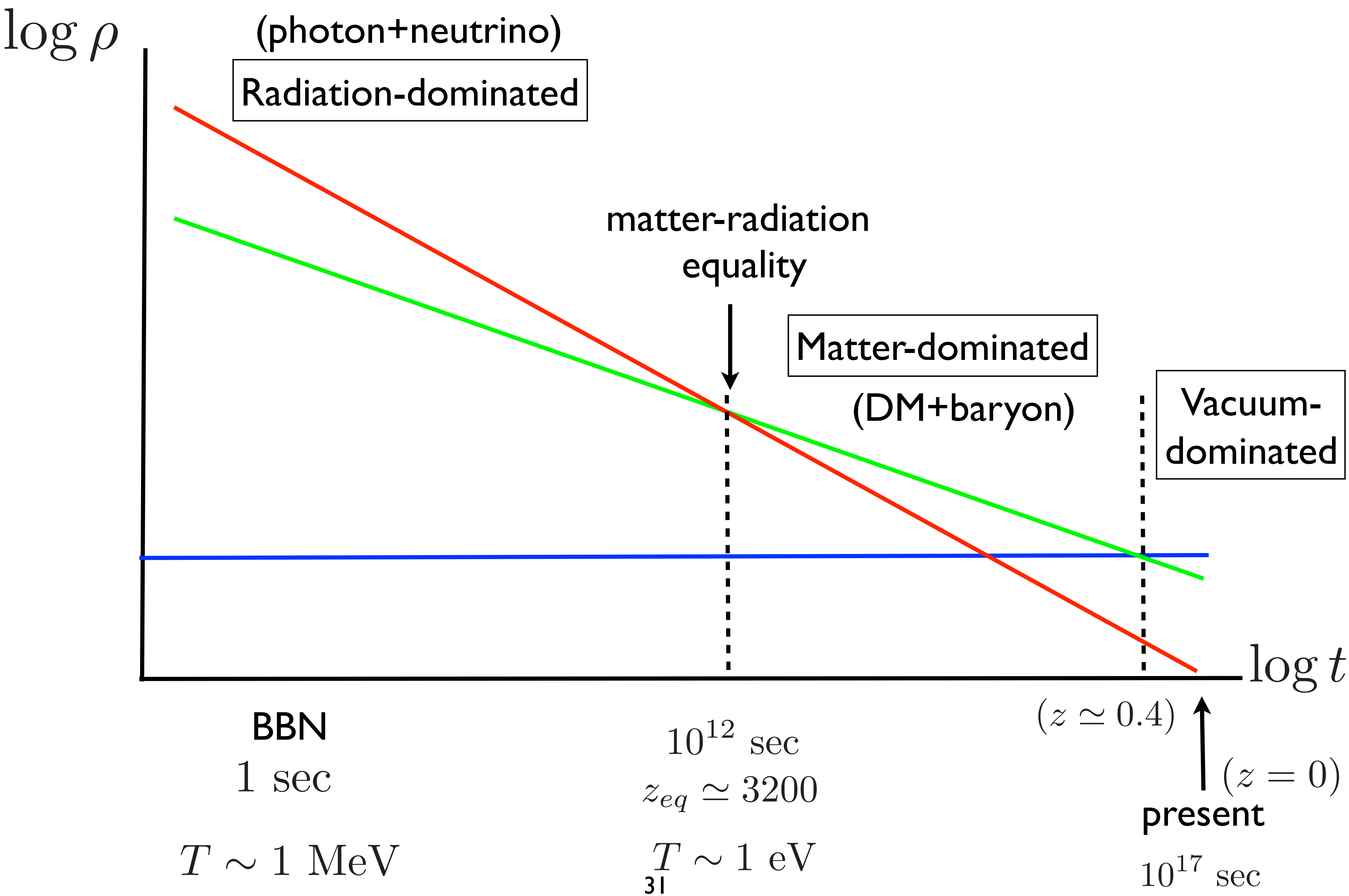
Total density $$\Omega = \sum_i \Omega_i = \frac{\sum_i \rho_i}{\rho_c}$$

Local geometry is related to the density in the Universe

Closed	$k > 0,$	$\Omega > 1$
Flat	$k = 0,$	$\Omega = 1$
Open	$k < 0,$	$\Omega < 1$



- Evolution of energy density



At present time, $\rho_c = 3H_0^2 M_P^2 = 1.88 \times 10^{-29} \mathrm{g\,cm}^{-3}$

Photons are in the Planck distribution and the energy density is

$$\rho_{\gamma,0} = a_B T_{\gamma,0}^4 = 4.64 \times 10^{-34}\,\mathrm{g\,cm}^{-3}$$

Radiation energy constant $a_B = \dfrac{8\pi^5 k_B^4}{15h^3c^3} = 7.5657 \times 10^{-15}\, erg\, cm^{-3}\, deg^{-4}$

Therefore $$\boxed{\Omega_{\gamma,0} \simeq 5 \times 10^{-5}}$$

Baryons are 5×10^{-9} numbers compared to the photons from BBN.

$$\rho_B = m_N N_B \simeq 938\,\mathrm{MeV} \times 5 \times 10^{-9} n_\gamma$$

$$n_\gamma = 421\ \mathrm{cm}^{-3}$$

Therefore $$\boxed{\Omega_B \simeq 0.04}$$

$\rho \propto R^{-4}$ Radiation

$\rho \propto R^{-3}$ (Cold) Matter

$\rho \propto \mathrm{const}$ Vacuum energy

At present time,

$$\rho_R^0 = 5 \times 10^{-5} \rho_c^0$$

$$\rho_M^0 = \rho_{DM}^0 + \rho_B^0 = 0.26 \rho_c^0$$

$$\rho_{vac}^0 = 0.74 \rho_c^0$$

We can go backward in time with values at present.

Matter density = Vacuum density at

$$\rho_M = \rho_M^0 \left(\frac{R}{R_0} \right)^{-3} = \rho_v = \rho_v^0 \quad \Rightarrow \quad \frac{1}{1+z} = \frac{R}{R_0} = \left(\frac{0.74}{0.26} \right)^{-1/3} \simeq 0.7$$

or $z \simeq 0.4$

Why does the Vacuum energy dominates recently?

$\boxed{\text{Coincidence problem}}$

$$\rho \propto R^{-4}$$ Radiation

$$\rho \propto R^{-3}$$ (Cold) Matter

$$\rho \propto \mathrm{const}$$ Vacuum energy

At present time,

$$\rho_R^0 = 5 \times 10^{-5} \rho_c^0$$

$$\rho_M^0 = \rho_{DM}^0 + \rho_B^0 = 0.26 \rho_c^0$$

$$\rho_{vac}^0 = 0.74 \rho_c^0$$

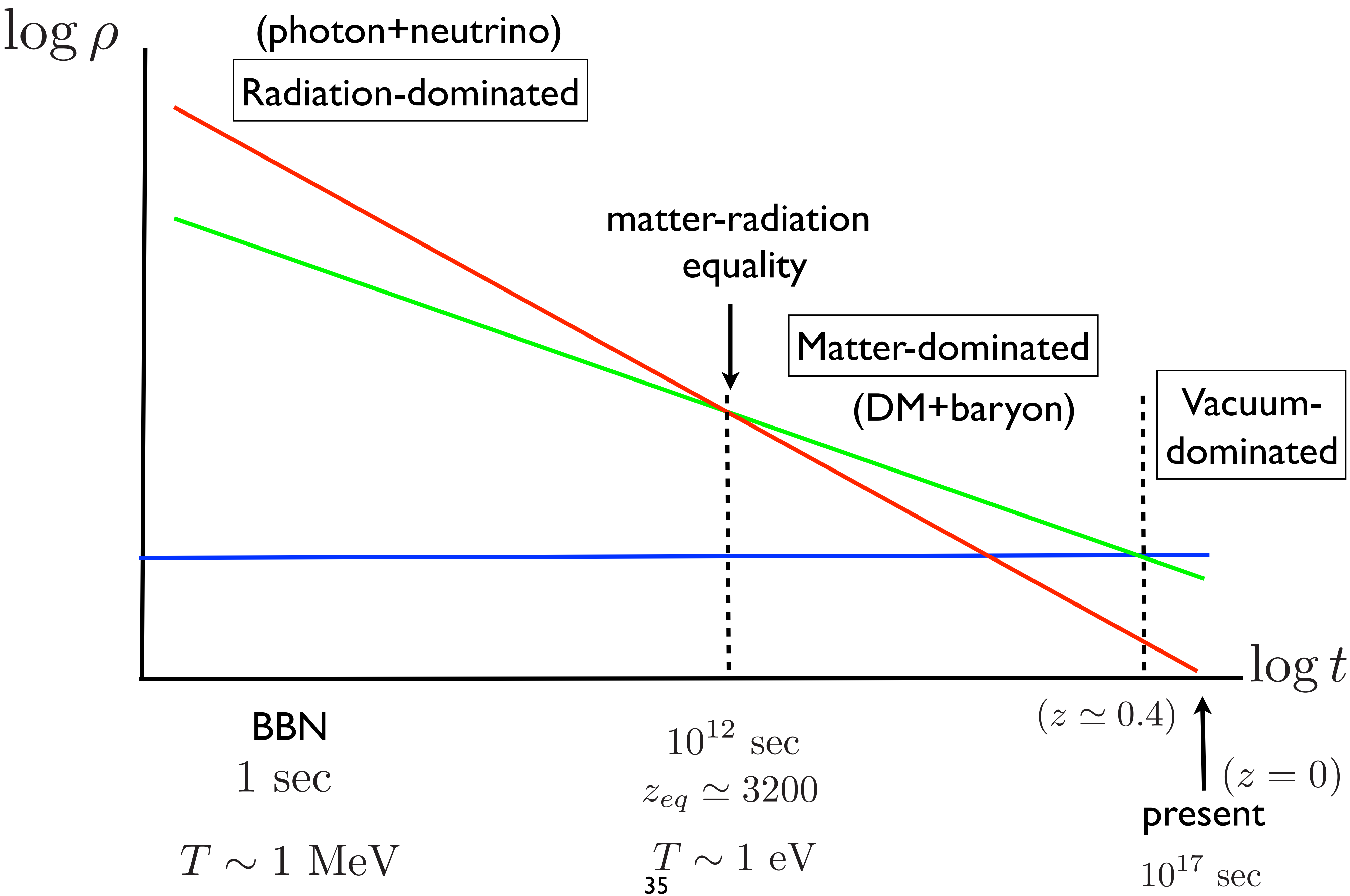
Radiation density = Matter density at

$$\left(\frac{R}{R_0}\right)^{-1} = \frac{\rho_M^0}{\rho_R^0} = \frac{0.26}{5 \times 10^{-5}} \simeq 5000 = 1 + z$$

If we include neutrino in the relativistic particles,

$$z_{eq} \simeq 3200 \quad \text{(70,000 years)}$$

- Evolution of energy density



Homogeneous and Isotropic Universe : FRW metric and Perfect fluid

$$g_{\mu\nu} = (1, -R^2(t), -R^2(t), -R^2(t))$$
$$g^{\mu\nu} = (1, -R^{-2}(t), R^{-2}(t), -R^{-2}(t))$$

$$\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - p g_{\mu\nu}$$

$$\boxed{H^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho}$$

$$\boxed{\dot{\rho} + 3H(\rho + p) = 0}$$

For k=0 (flat), and $p = \omega\rho$ we obtain $R \propto t^{2/3(1+\omega)}$

Radiation $(p = \frac{1}{3}\rho)$	$R \propto t^{1/2}$
Matter $(p = 0)$	$R \propto t^{2/3}$
Vacuum energy $(p = -\rho)$	$R \propto \exp(Ht)$

energy density

$$\rho \propto R^{-3(1+\omega)} \left(\begin{array}{lll} \rho \propto R^{-4} & \text{Radiation} & (p = \frac{1}{3}\rho) \\ \rho \propto R^{-3} & \text{(Cold) Matter} & (p = 0) \\ \rho \propto \text{const} & \text{Vacuum energy} & (p = -\rho) \end{array} \right.$$

(cosmological constant)

particle horizon

$d_p(t) = 2t$	Radiation $(p = \frac{1}{3}\rho)$
$d_p(t) = 3t$	Matter $(p = 0)$

age of universe

$$t_0 = \frac{2}{3(1+\omega)} H_0^{-1}$$

$t_0 = \frac{1}{2} H_0^{-1}$	Radiation $(p = \frac{1}{3}\rho)$
$t_0 = \frac{2}{3} H_0^{-1}$	Matter $(p = 0)$