Axion Cosmology

Lecture 2 | Axions as Dark Matter

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Outline

Peccei-Quinn symmetry breaking and axion potential

Axions or axion-like particles as dark matter candidates

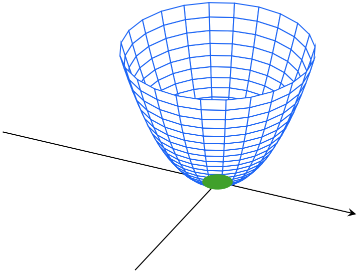
Structure formation with ALPs

Where does the axion come from? Peccei-Quinn Mechanism

- Add to the Standard Model a complex scalar field φ_{PQ}.
- Assume a U(1)-symmetric potential exists for φ_{PQ}:

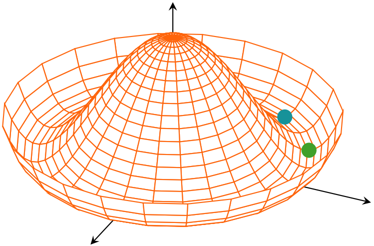
$$V_{\text{PQ}}(\varphi_{\text{PQ}}) = V_{\text{PQ}}(|\varphi_{\text{PQ}}|) \quad , \quad \text{U}(1) : \varphi_{\text{PQ}} \mapsto \varphi_{\text{PQ}} e^{i\alpha}.$$

- The vacuum state is determined by the shape of the potential.



$\langle \varphi_{\text{PQ}} \rangle = 0$

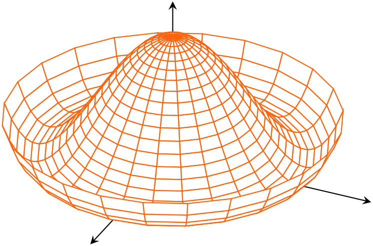
invariant under U(1)



$\langle \varphi_{\text{PQ}} \rangle = \chi e^{i\alpha},\ \alpha \in [0, 2\pi)$

not-invariant under U(1)

Spontaneous symmetry breaking



Assume that in the early universe, a phase transition turns the unbroken potential to broken potential. In the broken phase

angular mode (axion)

$$\varphi_{\mathrm{PQ}} = \chi \exp\left\{ \frac{i\,\phi}{f_\phi} \right\} \equiv \chi \exp\left\{ i\,\theta \right\}$$

radial mode

axion decay constant

Axion potential and the axion mass

Around the QCD phase transition, non-perturbative effects generate a potential for the axion $V(\theta)$ that is minimized at $\theta = 0$: 1606.07494

$$V(\theta) \approx \begin{cases} m_\phi^2(T) f_\phi^2 [1 - \cos(\theta)] \propto T^{-8.16}, & T \gg \Lambda_{\mathrm{QCD}} \simeq 150\,\mathrm{MeV} \\ -m_\pi^2 f_\pi^2 \sqrt{1 - \dfrac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\dfrac{\theta}{2}\right)}, & T \ll \Lambda_{\mathrm{QCD}} \end{cases}$$

In summary, the Peccei-Quinn mechanism turns the constant θ parameter into a dynamical one, and $\theta = 0$ is explained by system dynamics.

Axion and axion-like-particle (ALP)

In order axion to solve the Strong CP problem, the axion mass m_ϕ and the axion decay constant f_ϕ should be related to each other.

$$m_\phi = \frac{\sqrt{m_u m_d}}{m_u + m_d} \frac{m_\pi f_\pi}{f_\phi} \simeq 5.7 \times \left(\frac{10^{12}\,\mathrm{GeV}}{f_\phi} \right) \mu\mathrm{eV}.$$

An axion for which this relation does not hold is known as an axion-like-particle (ALP).

Cosmological evolution of a ALP field

Start with the action for the ALP field (neglect Standard Model interactions):

$$S = \int d^4x \sqrt{-g}[-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)].$$

Take the background geometry to be the FRWL geometry with the metric

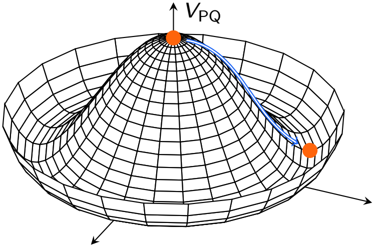
$$ds^2 = -\,dt^2 + a^2(t)\delta_{ij}\,dx^i\,dx^j$$

Then, the ALP field obeys the following equation of motion:

$$\ddot{\theta} + 3H\dot{\theta} - \frac{\nabla^2}{a^2}\theta + \frac{1}{f_\phi^2}\frac{dV}{d\theta} = 0$$

This is a second order differential which can be solved after specifying the initial conditions.

Initial conditions: Pre- and post-inflationary scenario



Post-inflationary scenario:

- Different initial angle in each Hubble patch.
- Inhomogeneous including topological defects.

Pre-inflationary scenario:

- Random initial angle, $\theta \in [-\pi, \pi)$, in the observable universe.
- Initially homogeneous without topological defects.

ALP evolution in the pre-inflationary scenario

By assuming homogeneity, we can neglect the laplacian term $\nabla^2\theta$ in the equation of motion. Then, we get a second-order ODE:

$$\ddot{\theta} + 3H\dot{\theta} + \frac{1}{f_\phi^2}\frac{dV}{d\theta} = 0.$$

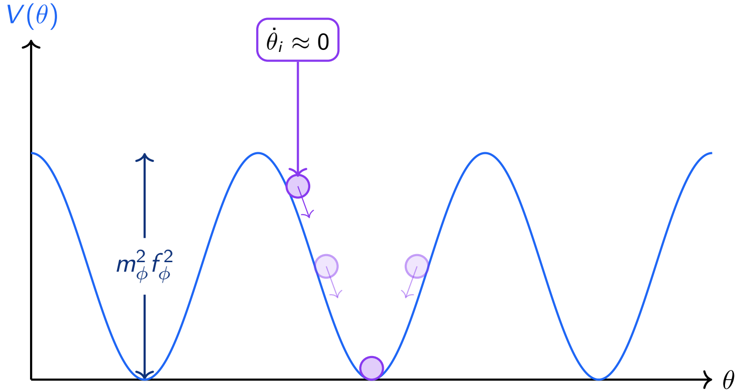
Assume a temperature-independent mass, and small angle approximation:

$$V(\theta) = m_\phi^2 f_\phi^2[1 - \cos\theta] \overset{\theta \ll 1}{\approx} \frac{1}{2} m_\phi^2 f_\phi^2 \theta^2.$$

This way, we obtain the eom of a damped harmonic oscillator:

$$\ddot{\theta} + \underbrace{3H}_{\text{Hubble friction}}\dot{\theta} + \underbrace{m_\phi^2}_{\text{Oscillation frequency}}\theta = 0$$

ALP evolution in radiation domination (cartoon)



ALP evolution in radiation domination

In order ALPs to be dark matter, they need to created well before matter-radiation equality. Assuming a radiation dominated era[1]

$$\rho = 3M_{\rm pl}^2 H^2 \propto a^{-4} \implies H = \frac{1}{2t}$$

the equation of motion for θ can be solved exactly. With $\dot{\theta}_i = 0$

Gamma function

Bessel function

$$\theta(t) = \theta_i \, 2^{1/4}\, \Gamma\left(\frac{5}{4}\right) \frac{J_{1/4}(m_\phi t)}{(m_\phi t)^{1/4}}$$

Initial angle

If we look to the early- and late-time limits

$$\theta(t) \propto \begin{cases} \text{constant} & H \gg m_\phi \\ a(t)^{-3/2} \cos(m_\phi t + \alpha) & H \ll m_\phi \end{cases}$$

[1] and neglecting the changes in the effective degrees of freedom

Evolution of the ALP energy density

The energy density and the pressure of the ALP is given respectively by

$$\rho = \frac{1}{2} f_\phi^2 \dot{\theta}^2 + \frac{1}{2} m_\phi^2 f_\phi^2 \theta^2 \quad \text{and} \quad P = \frac{1}{2} f_\phi^2 \dot{\theta}^2 - \frac{1}{2} m_\phi^2 f_\phi^2 .$$

At early times, the ALP field is frozen:

$$\rho \approx \frac{1}{2} m_\phi^2 f_\phi^2 \theta_i^2, \qquad P \approx -\rho$$

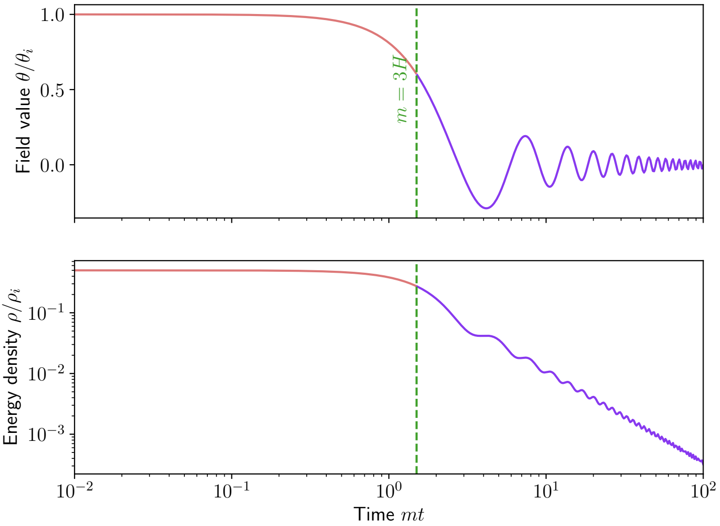
ALP behaves as dark energy

At late times, the ALP field is oscillating:

$$\rho \approx \sqrt{m_\phi} f_\phi^2 \frac{\sqrt{2}}{\pi} \left(\Gamma\left(\frac{5}{4}\right) \theta_i \right)^2 t^{-3/2} \propto a^{-3}, \quad P \approx 0$$

ALP behaves as dark matter

Plot of the evolution of the ALP energy density



ALP energy density today

The late-time ALP energy density is commonly expressed as

$$\rho(a) \approx \underbrace{\rho(a_{\rm osc})}_{\approx \rho_i} \left(\frac{a_{\rm osc}}{a}\right)^3 \approx \frac{1}{2} m_\phi^2 f_\phi^2 \theta_i^2 \left(\frac{a_{\rm osc}}{a}\right)^3 = \frac{1}{2} m_\phi^2 f_\phi^2 \theta_i^2 \frac{g_{\star,s}(T_0)}{g_{\star,\rm osc}} \left(\frac{T_0}{T_{\rm osc}}\right)^3,$$

entropy conservation

where $T_{\rm osc}$ is the temperature at the onset of oscillations. Usually, it is defined by

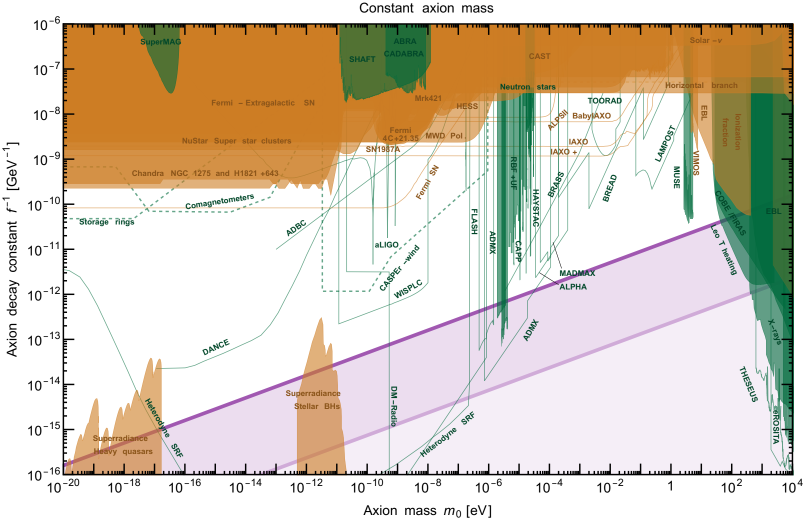
$$m_\phi(T_{\rm osc}) = 3H(T_{\rm osc}).$$

After some work, we find the today's relic density as 2403.17697

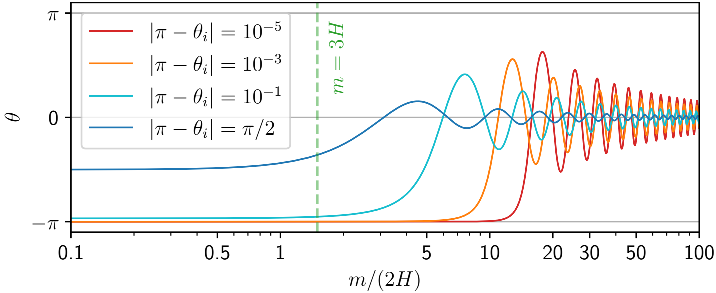
$$h^2 \Omega_{\phi,0} \approx 0.12 \left(\frac{m_\phi}{10^{-21}\,\rm eV}\right)^{1/2} \left(\frac{f_\phi}{5\times 10^{16}\,\rm GeV}\right)^2 \theta_i^2.$$

For a given m_ϕ and f_ϕ the initial angle θ_i yielding the correct amount of dark matter can be found.

ALP dark matter in standard misalignment



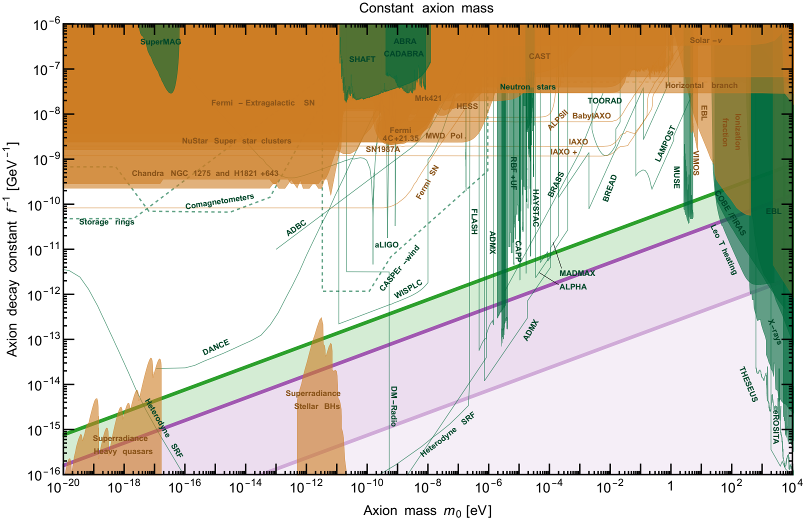
Anharmonicity corrections



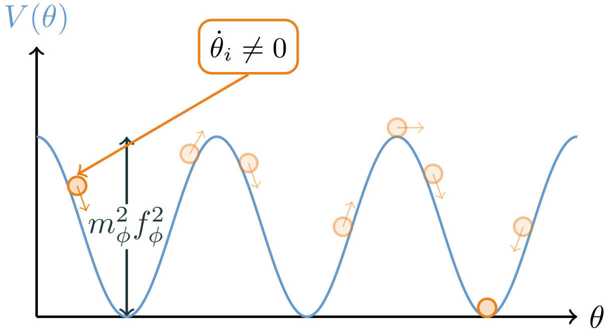
If the initial angle is very close to $\pm\pi$, the onset of oscillations dot delayed due to the tiny potential gradient at the top: 1909.11665

$$\frac{m_\phi}{2H_{\rm osc}} = \ln\left[\frac{1}{|\pi - \theta_i|}\frac{2^{1/4}\pi^{1/2}}{\Gamma(5.4)}\right]$$

ALP dark matter in large misalignment



Kinetic misalignment



- Start with a large kinetic energy, so the field overcomes many barriers before stopping. 1910.14152, 1911.11885
- The onset of oscillations can be significantly delayed.

Dark matter from QCD axion

Since the mass of the QCD axion depends on the temperature, the energy density does not scale as $\rho \propto a^{-3}$ after the onset of oscillations. However, number density does:

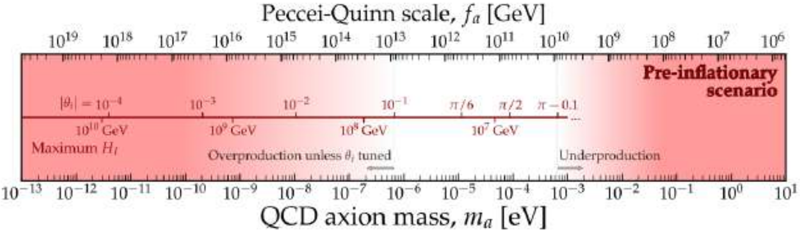
$$n_\phi(T_0) = n_\phi(T_{\rm osc})\left(\frac{a_{\rm osc}}{a_0}\right)^3.$$

If the QCD axion starts oscillating before the QCD phase transition, when $f_\phi \gtrsim 3 \times 10^{15}\,\mathrm{GeV}$, the relic density is

$$h^2\Omega_\phi = 0.12\left(\frac{\theta_i}{2.155}\right)^2\left(\frac{28\,\mu\mathrm{eV}}{m_{\phi,0}}\right)^{1.16}.$$

For a given QCD axion mass $m_{\phi,0}$, the initial angle yielding the correct dark matter density can be found.

QCD axion dark matter window



QCD axion dark matter in the post-inflationary scenario

In the post-inflationary scenario, universe will have different patches where the misalignment angle is different. We can estimate an effective angle by

$$\left\langle \theta_i^2 \right\rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\theta_i \, \theta_i^2 = \left(\frac{\pi}{\sqrt{3}} \right)^2 = (1.81)^2.$$

Due to the anharmonicities in the potential, the actual value is somewhat larger: 1511.02867

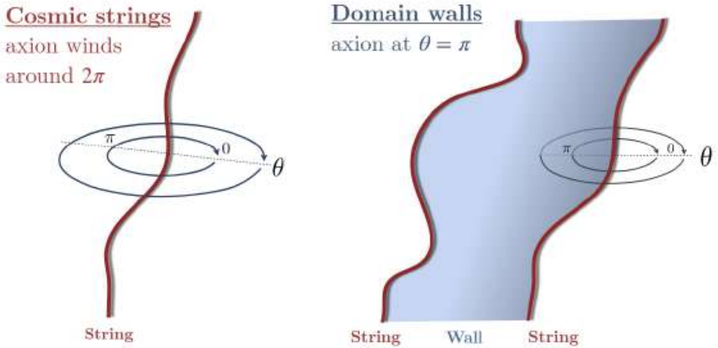
$$\left\langle \theta_i \right\rangle^2 = (2.155)^2.$$

Combining this with our previous result yields

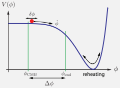
$$h^2 \Omega_\phi = 0.12 \left(\frac{28\,\mu\text{eV}}{m_{\phi,0}} \right)^{1.16}.$$

Demanding correct dark matter density gives us a prediction for the QCD axion mass!!! It does not!.

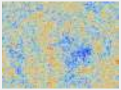
Topological defects



Brief history of structure formation

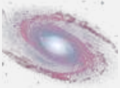


1. Inflationary perturbations and reheating



2. Temperature fluctuations in the thermal plasma

3. Dark matter density fluctuations



6. Formation of stars and galaxies

5. Collapse of baryons in DM halos

4. Collapse of dark matter fluctuations

From inflation to power spectrum

The distribution of all components in the universe are approximately homogeneous and isotropic. The deviations from the homogeneity are represented by the density δ and velocity θ perturbations:

$$\begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\theta} \end{bmatrix} = [\delta_\gamma, \delta_c, \delta_b, \delta_\nu, \ldots, \theta_\gamma, \theta_c, \theta_b, \theta_\nu, \ldots]^T$$

The time evolution of these perturbations can be calculated using cosmological perturbation theory: CLASS CAMB

$$\frac{d}{dt}\begin{bmatrix} \boldsymbol{\delta}(t) \\ \boldsymbol{\theta}(t) \end{bmatrix} = \begin{pmatrix} \cdot & \cdots & \cdot \\ \vdots & \ddots & \vdots \\ \cdot & \cdots & \cdot \end{pmatrix} \begin{bmatrix} \boldsymbol{\delta}(t) \\ \boldsymbol{\theta}(t) \end{bmatrix}, \qquad \text{Inflation model} \Rightarrow \begin{bmatrix} \boldsymbol{\delta}(t=0) \\ \boldsymbol{\theta}(t=0) \end{bmatrix}$$

Matter power spectrum

The correlation between the matter energy density perturbations is

$$\langle \delta_m(\mathbf{k}, t)\delta_m(\mathbf{k}', t)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k} - \mathbf{k}')\, P_m(k, t)$$

Power spectrum

The power spectrum can be expressed conveniently as

$$P_m(k, t) = \frac{2\pi^2}{k^3}\,\mathcal{P}_{\rm inf}(k)\; T^2(k, t)$$

initial conditions

subsequent evolution

Initial conditions are imprinted by inflation:

$$\mathcal{P}_{\rm inf}(k) = A_s\left(\frac{k}{k_\star}\right)^{n_s-1}, \quad A_s \simeq 2.1\times 10^{-9}, \quad k_\star = 0.05\,{\rm Mpc}^{-1}.$$

Constraints on the matter power spectrum

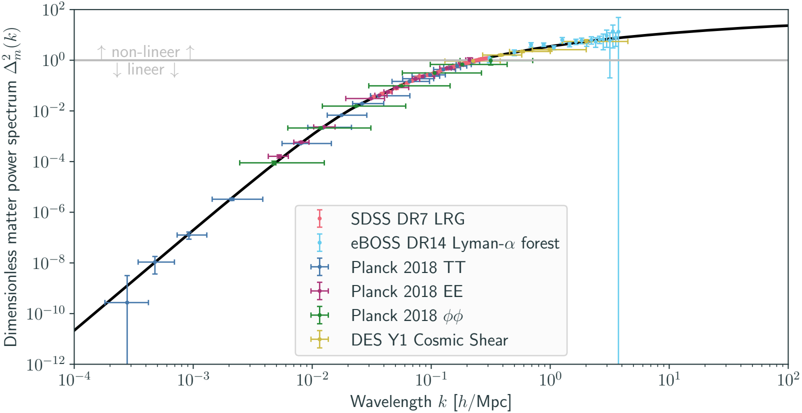


Figure 1: For the experimental data, see this GitHub repo.

ALP fluctuations and the mode functions

Even in the pre-inflationary scenario ALP field has some fluctuations on top of the homogeneous background, which can be desribed by the mode functions in Fourier space:

$$\theta(t, \mathbf{x}) = \underbrace{\Theta(t)}_{\text{homogeneous background}} + \int \frac{d^3k}{(2\pi)^3} \underbrace{\theta_k}_{\text{mode functions}} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}$$

These fluctuations are seeded by adiabatic and/or isocurvature perturbations:

- Adiabatic perturbations are due to the perturbations in the radiation bath. They are unavoidable.
- If ALPs exist during inflation and light, $m \ll H_{\text{inf}}$, they pick up quantum fluctuations. According to CMB, they should be small.

Equation of motion for the mode functions

The equation of motion for the mode functions can be derived from the perturbed FLRW metric:

$$ds^2 = -\left[1 - 2\,\Phi(t,\mathbf{x})\right] + a^2(t)\left[1 + 2\,\Phi(t,\mathbf{x})\right]\delta_{ij}\,dx^i\,dx^j\,.$$

curvature perturbations

In radiation era, curvature Fourier modes Φ_k has an exact solution.

$$\Phi_k(t) = 3\,\Phi_k(0)\left[\frac{\sin t_k - t_k \cos t_k}{t_k^3}\right], \quad t_k = \frac{k/a}{\sqrt{3}H},$$

imprinted by inflation

while the equation of motion for the ALP mode functions is

$$\ddot{\theta}_k + 3H\dot{\theta}_k + \left(\frac{k^2}{a^2} + m_\phi^2 \cos\Theta\right)\theta_k = 2\Phi_k m_\phi^2 \sin\Theta - 4\dot{\Phi}_k\dot{\Theta}$$

effective frequency

sources the fluctuations

ALP dark matter power spectrum

The size of fluctuations is determined by the density contrast:

$$\delta_{\rho_\theta} \equiv \frac{\rho_\theta(t, \mathbf{x}) - \bar{\rho}_\theta(t)}{\bar{\rho}_\theta(t)}.$$

For the ALP field, the Fourier modes of this density contrast are

$$\delta_{\rho_\theta}(t, k) = 2\left[\frac{\dot{\Theta}\dot{\theta}_k}{m_\phi^2} + \Phi_k \frac{\dot{\Theta}^2}{m_\phi^2} + \sin\Theta\theta_k\right].$$

From this, we can determine the power spectrum:

$$\mathcal{P}_{\rho_\theta}(k) = \frac{k^3}{2\pi^2}\left\langle \left|\delta_{\rho_\theta}(k, t)\right|^2 \right\rangle.$$

Evolution of the density contrast at late times

We can use an effective description using the WKB approximation:
1207.3124

$$\Theta(t) = a^{-3/2}[\Theta_+ \cos(mt) + \Theta_- \sin(mt)]$$
$$\theta_k(t) = \theta_+(k,t)\cos(mt) + \theta_-(k,t)\sin(mt)$$

The evolution of the density contrast for sub-horizon modes, $k/a \gg H$, obeys the following differential equation both in matter and radiation when baryons are neglected:

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \left(c_{s,\mathrm{eff}}^2 \frac{k^2}{a^2} - 4\pi G_N \bar{\rho}_\Theta \right)\delta_k = 0, \quad c_{s,\mathrm{eff}}^2 \approx \frac{1}{4}\frac{k^2}{a^2 m^2}.$$

effective sound speed — gravitational instability

The evolution for Cold Dark Matter (CDM) is recovered in the limit

$$c_s^2 \to 0 \quad \text{or} \quad m \to \infty$$

Axion Jeans scale

For sub-horizon $k/a \gg H$ and non-relativistic $k/a \ll m$ modes, the density contrast evolution is

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \left(\frac{1}{4}\frac{(k/a)^4}{m^2} - 4\pi G_N \bar{\rho}_\Theta \right)\delta_k = 0$$

"quantum" pressure

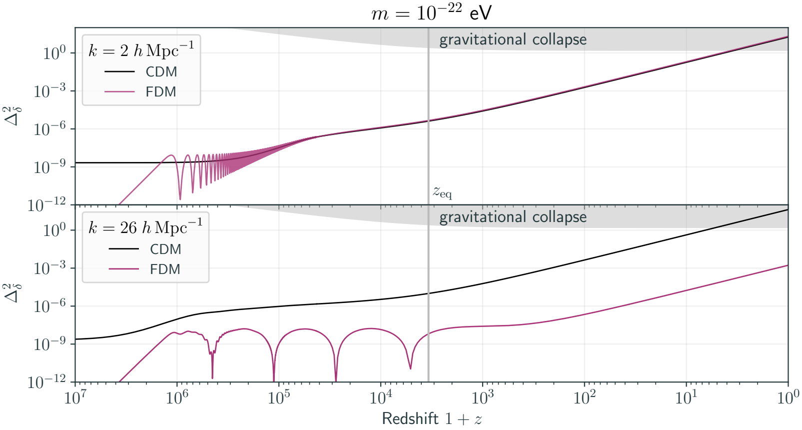
gravitational instability

The scale at which the "quantum" pressure and gravitational instability gravitational instability becomes equal is called the axion Jeans scale:

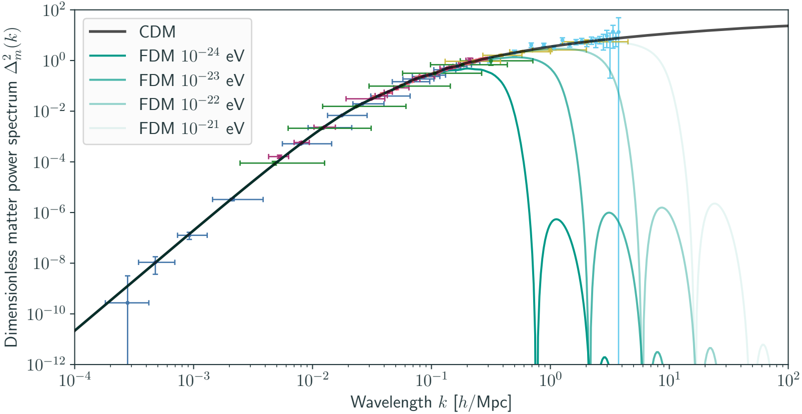
$$k_J(a) = (16\pi G_N a \bar{\rho}_\theta)^{1/4}\sqrt{m} = 66.5 \times a^{1/4}\left(\frac{h^2\Omega_\theta}{h^2\Omega_{\rm DM}}\right)\left(\frac{m}{10^{-22}\,\text{eV}}\right)\text{Mpc}^{-1}$$

- Modes above the Jeans scale oscillate with a frequency given by the effective sound speed both in matter- and radiation-domination.
- Modes below the Jeans scale behaves like CDM. They grow logarithmically during the radiation era, and linearly during the matter era.

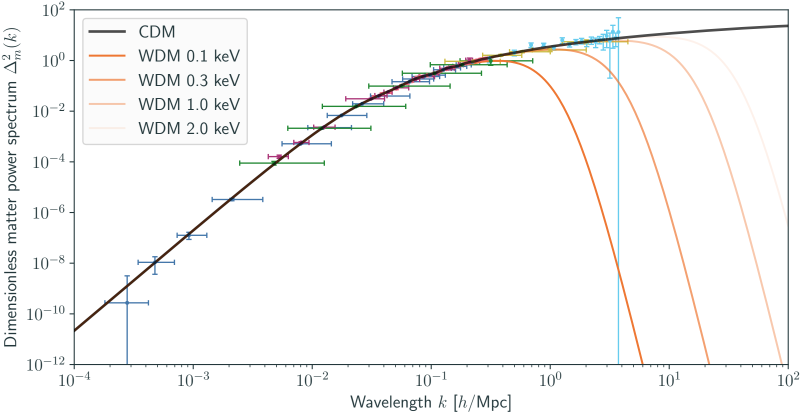
Density contrast evolution



Linearly processed power spectrum: ALPs



Linearly processed power spectrum: Warm dark matter



Suppression of the small-scale structure

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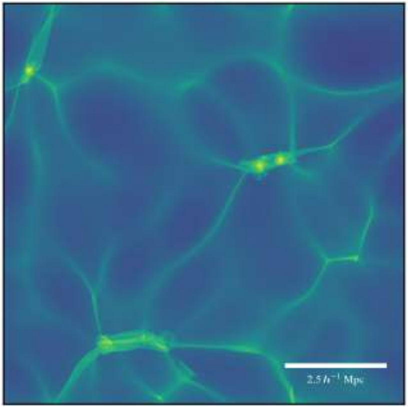
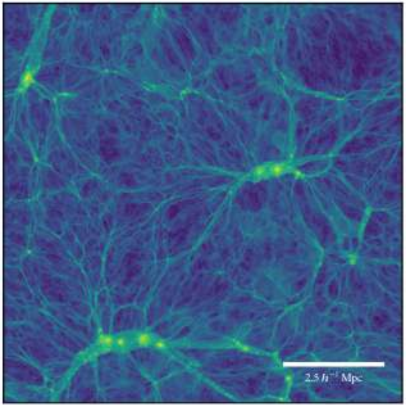


ALP with $m = 7 \times 10^{-23}\,\text{eV}$



Cold Dark Matter

Density profiles of dark matter halos

Useful parameters to describe the dark matter halos:

$$\underbrace{\left.\frac{\partial \ln \rho(r)}{\partial \ln r}\right|_{r=r_s} = -2}_{\text{scale radius}}, \quad \underbrace{\rho_s = \rho(r = r_s)}_{\text{scale density}}, \quad \underbrace{M_s = 16\pi \rho_s r_s^3 \left(\ln 2 - \frac{1}{2}\right)}_{\text{scale mass}}.$$

In order to determine these parameters, we need to know the density profile:

- On all scales, the profile is well-approximated by the NFW profile.
  astro-ph/9611107

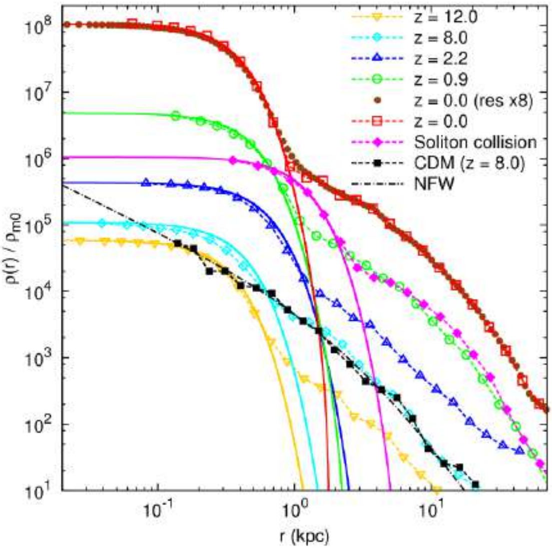
$$\rho_{\rm NFW}(r) = \frac{4\rho_s}{(r/r_s)(1 + \frac{r}{r_s})^2}$$

- For ALPs, the profile is scale-dependent. At large scales the profile is NFW, while at small scales halos have a soliton profile. 1406.6586

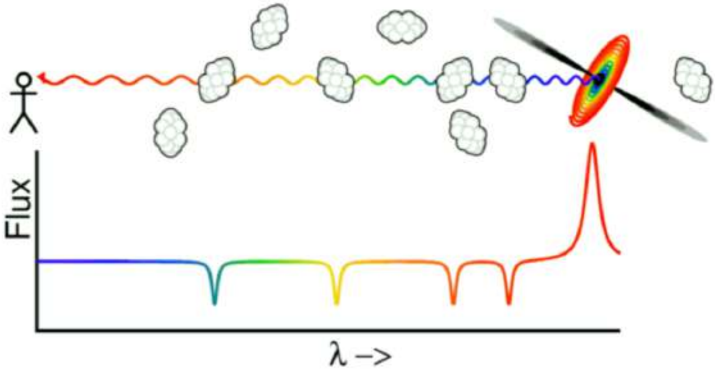
$$\rho_{\rm sol}(r) \approx \frac{2.9\rho_s}{\left(1 + \left(r/\sqrt{7}r_s\right)^2\right)^8} \implies \rho_s \propto m^6 M_s^4$$

ALP Halo Profiles

1407.7762



Lyman-alpha forest



The Lyman-alpha forest is a popular tool to measure the matter power spectrum at small scales. It puts a strong bound on the ALP mass given by $m > 2 \times 10^{-20}$ eV (95% C.L.). 2007.12705