Axion Cosmology

Lecture 1 | A Brief Tour of Cosmology

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Cosmological principle

All of modern cosmology is based on the following assumption supported strongly by data:

The universe is **homogeneous** and **isotropic** when averaged on sufficiently large (more than 100 Mpc) scales.

- Homogeneity means that the universe looks the same at every location.
- Isotropy means that the universe looks the same at every direction.
- On the scales of the Milky Way, clearly the universe is neither homogeneous nor isotropic.

Friedmann–Lemaître–Robertson–Walker (FLRW) metric

FLRW metric is the unique metric which is consistent with the symmetries of a homogeneous and isotropic universe:

$$ds^2 = -\,dt^2 + \underset{\substack{\uparrow \\ \text{scale factor}}}{a^2(t)} \left[\frac{dr^2}{1 - \underset{\substack{\uparrow \\ \text{curvature of the spatial slices}}}{K}\, r^2/R_0^2} + r^2\, d\Omega^2 \right]$$

- The scale factor determines the expansion of the universe.
- Curvature of the spatial slices determines whether the universe is flat ($K = 0$), closed ($K = 1$), or open ($K = -1$).
- Observations strongly suggest a flat universe, $K = 0$. astro-ph/0004404

$$ds^2 = -\,dt^2 + a^2(t)\big[dx^2 + dy^2 + dz^2\big] = -\,dt^2 + a^2(t)\delta_{ij}\, dx^i\, dx^j$$

Friedmann's equations

The evolution of the universe is governed by the Einstein equations:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Geometry (FLRW)

"Stuff" in the universe

Homogeneity and isotropy requires that all species are described by perfect fluids:

$$T_\mu^{\ \nu} = \mathrm{diag}\left(-\rho, P, P, P\right)$$

Energy density

Pressure

Then, the Einstein equations imply Friedmann's equations:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\rm pl}^2}\sum_i \rho_i \quad , \quad \frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm pl}^2}\sum_i (\rho_i + 3P_i)$$

Hubble parameter

Reduced Planck mass: $M_{\rm pl} = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV

Continuity equation

From Einstein's equations one can also derive the continuity equation:

$$\dot{\rho} + 3H(\rho + P) = 0 \overset{\text{perfect fluid}}{\Longrightarrow} \rho \propto a^{-3(1+w)}, \quad \boxed{w} = \frac{P}{\rho}$$

equation of state

For typical species in the universe:

- Pressureless matter (Cold dark matter, baryons at late times): $w = 0 \implies \rho \propto a^{-3}$.
- Radiation (Photons, massless neutrinos): $w = 1/3 \implies \rho \propto a^{-4}$.
- Cosmological constant or dark energy: $w = -1 \implies \rho = \text{constant}$.

Redshift

The light from distant galaxies are redshifted due to the expansion of the universe:

$$\overbrace{\lambda_{\text{emit}}}^{\text{Wavelength when emitted}} \rightarrow \underbrace{\lambda_{\text{obs}}}_{\text{Wavelength when observed}} = \frac{\lambda_{\text{emit}}}{a_{\text{emit}}} \quad , \quad z_{\text{emit}} = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{\text{us}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} \implies \frac{1}{a} = 1 + z$$

The parameter $z = 1 - a^{-1}$ is called the redshift and can be used as a time variable:

- $z = 0$ means today.
- $z \gg 1$ means long time ago.

Hubble parameter and Hubble tension

Today's value of the Hubble parameter H_0 was first measured by plotting redshift of a bunch of galaxies agains their distances from us. It's value is typically expressed as

$$H_0 \equiv \left.\frac{\dot{a}}{a}\right|_{z=0} = 100\,h\,\mathrm{km/s/Mpc} = 2.13 \times 10^{-33}\,h\,\mathrm{eV}.$$

They are two ways to measure the Hubble constant today:

- Like Hubble did with plotting redshift versus distance: 2112.04510

$$h = 0.73 \pm 0.01$$

- By best-fit to our CDM model: 1807.06209

$$h = 0.674 \pm 0.005$$

There is an almost 5σ discrepency between the two measurements. This is known as the Hubble tension.

The Hot Big Bang

If we reverse the expansion of the universe, eventually we reach to a singularity, commonly referred to as Big Bang. What happened in the universe since the Big Bang?

- Since the universe is expanding, it was denser and hotter in the past.
- The particles were colliding frequently and the universe was in a state of thermal equilibrium with an associated temperature T.
- In cosmology, it is convenient to set the Boltzmann's constant to unity and associate temperature with energy:

$$k_B \approx 8.62 \times 10^{-5}\,\mathrm{eV\,K^{-1}} \equiv 1 \implies 1\,\mathrm{eV} \approx 1.16 \times 10^{4}\,\mathrm{K}.$$

- A useful relation between the temperature T of the universe and the age of the universe (time since the Big Bang) is

$$\left(\frac{T}{1\,\mathrm{MeV}}\right) \approx \left(\frac{t}{1\,\mathrm{s}}\right)^{-1/2}.$$

Particles in thermal equilibrium

When particles are relativistic and in thermal equilibrium, their contribution to the energy density is very simple:

$$\rho = \frac{\pi^2}{30} g \, T^4 \times \begin{cases} 1 & \text{for bosons} \\ \frac{7}{8} & \text{for fermions} \end{cases}$$

number of degrees of freedom

They are two ways in which particles may decouple from the thermal bath:

- Chemical decoupling: Temperature drops below the mass of the particle, $m > T$. Particle becomes Boltzmann suppressed.

$$\rho \propto e^{-m/T}$$

- Kinetic decouplig: The interaction rate drops below the Hubble scale, $\Gamma < H$.

Energy density during radiation domination and entropy

The energy density in the radiation bath can be expressed as

$$\rho_{\text{rad}} = \frac{\pi^2}{30}\, g_\star(T)\, T^4, \quad g_\star(T) = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \sum_{\text{fermions}} \frac{7}{8} g_i \left(\frac{T_i}{T}\right)^4 .$$

effective dofs in energy density

By using basic thermodynamics we can also express the entropy as

$$s(T) = \frac{2\pi^2}{45}\, g_{\star,s}(T)\, T^3, \quad g_{\star,s}(T) = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \sum_{\text{fermions}} \frac{7}{8} g_i \left(\frac{T_i}{T}\right)^3 .$$

effective dofs in entropy

An important result is that the comoving entropy is conserved:

$$sa^3 = \text{constant} \implies g_{\star,s}(T)\, T^3 = \text{constant}$$

History of the universe

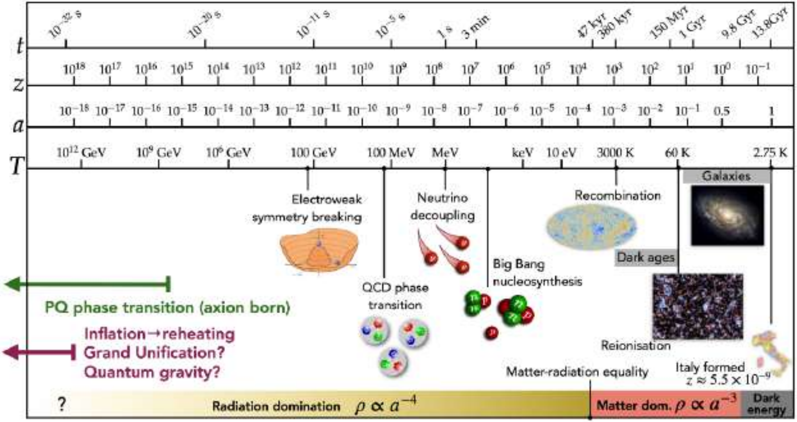
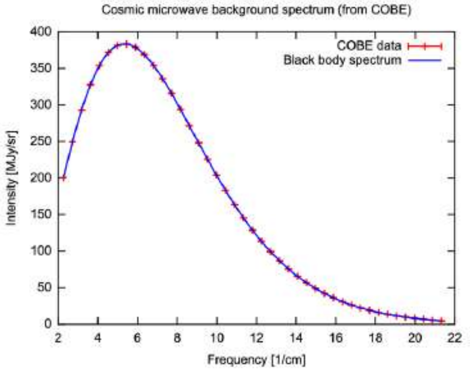


Figure 1: Taken from O'Hare's lectures 2403.17697

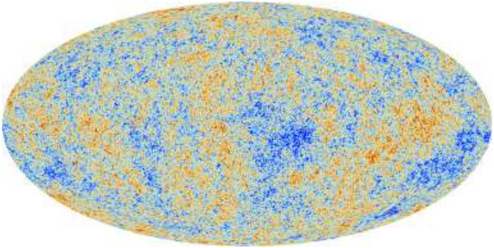
Cosmic Microwave Background (CMB) as a pure blackbody



The cosmic microwave background (CMB) radiation is an *almost* perfect blackbody with temperature astro-ph/9605054

$$T_{\mathsf{CMB}} = 2.728 \pm 0.004\,\mathsf{K}$$

Anisotropies in the CMB observed by Planck



CMB temperature map contains tiny temperature fluctuations. The correlation between these fluctuations can be expressed as an expansion in spherical harmonics:

$$\langle \delta T(\widehat{\mathbf{n}})\delta T(\widehat{\mathbf{n}}')\rangle = \sum_{l} \frac{2l+1}{4} \underbrace{C_l^{TT}}_{\text{expansion coefficients}} \underbrace{P_l(\widehat{\mathbf{n}}\cdot\widehat{\mathbf{n}}')}_{\text{Legendre polynomials}}$$

The coefficients C_l^{TT} can both be measured and calculated given a cosmological model, such as ΛCDM.

Best fit to ΛCMB from the Planck CMB data

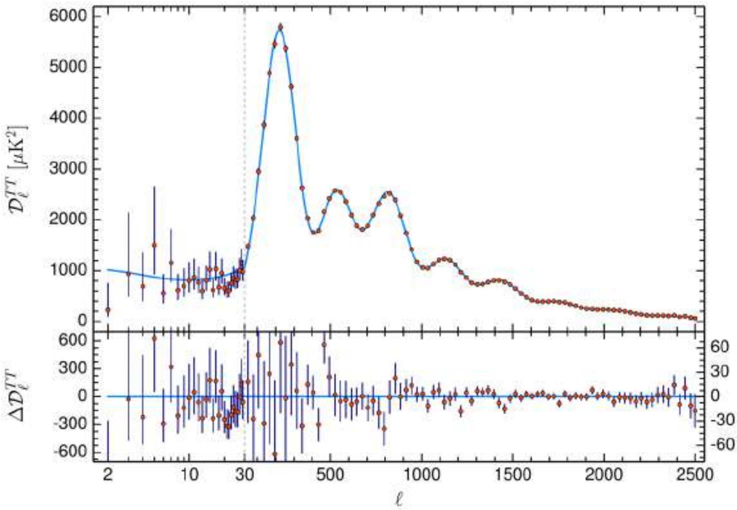


Figure 2: Measurements of $\mathcal{D}_l^{TT} = l(l+1)C_l^{TT}$ by Planck and best-fit to ΛCDM

1807.06209

Best fit to ΛCMB from the Planck CMB data

Quantity	Symbol	Value
Baryon density	$\Omega_b h^2$	0.02237 ± 0.00015
CDM density	$\Omega_c h^2$	0.1200 ± 0.0012
Primordial amplitude	$\ln\left(10^{1}0A_s\right)$	3.044 ± 0.014
Spectral index	n_s	0.9649 ± 0.0042
Hubble constant	h	67.36 ± 0.54
Matter density	Ω_m	0.3153 ± 0.0073

Energy densities are expressed in terms of their ratio to the critical energy density which is the required energy density for a flat universe:

$$\Omega_i = \frac{\rho_i}{\rho_{\text{crit}}} = \frac{\rho_i}{3M_{\text{pl}}^2 H_0^2}$$

Cold dark matter (and beyond?)

Properties of cold dark matter (CDM):

- A perfect fluid with no (or negligible) pressure.
- Non-gravitational interactions are absent or very weak.
- Stable

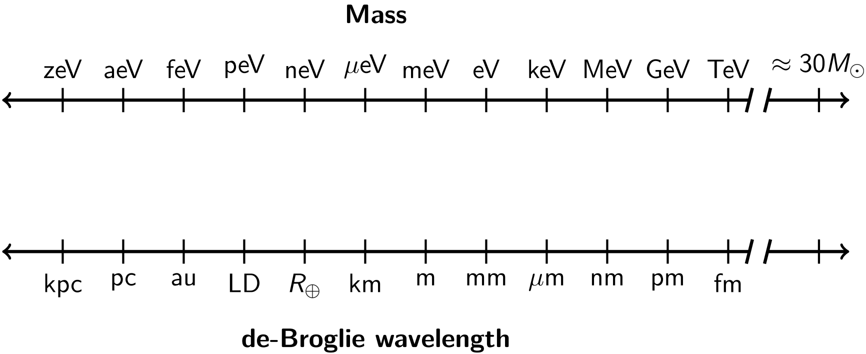
However CDM is too simple!

- It doesn't tell us anything about the underlying model.

CDM might also have some problems

- Small-scale crises (probably will be resolved by baryonic physics)
- Massive early galaxies observed by JWST
- Hubble tension
- ...

Candidates for dark matter



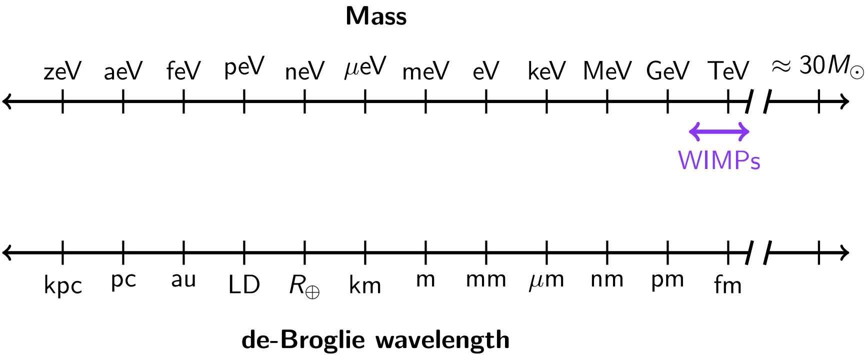
Mass

zeV aeV feV peV neV μeV meV eV keV MeV GeV TeV $\approx 30 M_\odot$

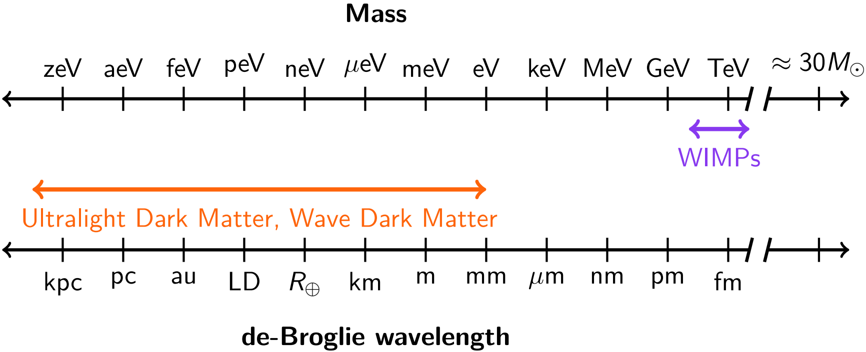
kpc pc au LD $R_\oplus$ km m mm μm nm pm fm

de-Broglie wavelength

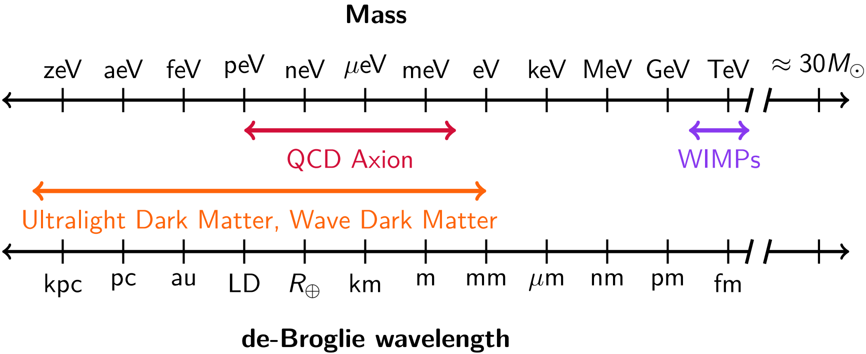
Candidates for dark matter



Candidates for dark matter



Candidates for dark matter



Mass

zeV aeV feV peV neV μeV meV eV keV MeV GeV TeV $\approx 30 M_\odot$

QCD Axion

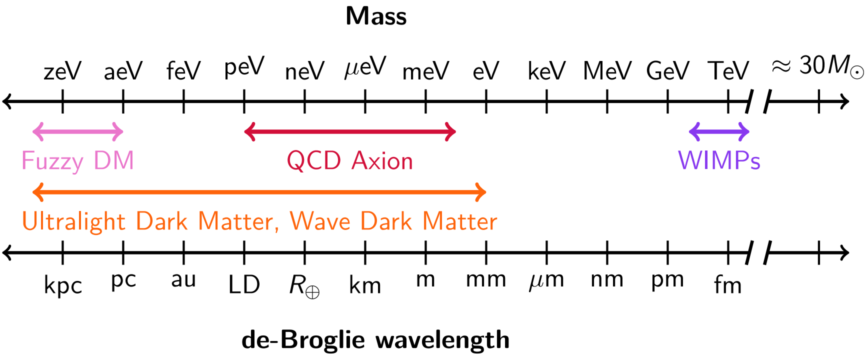
WIMPs

Ultralight Dark Matter, Wave Dark Matter

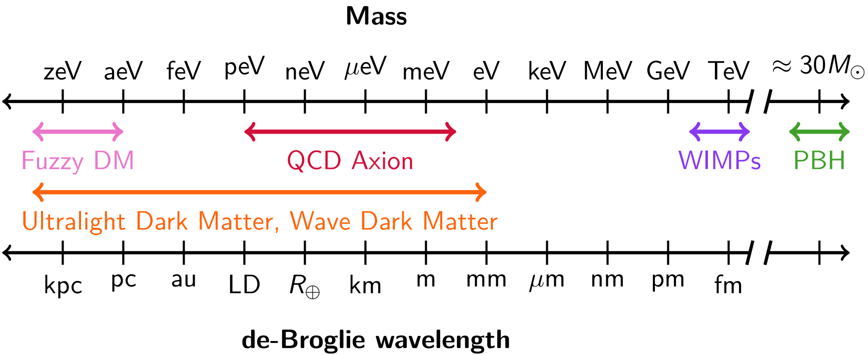
kpc pc au LD $R_\oplus$ km m mm μm nm pm fm

de-Broglie wavelength

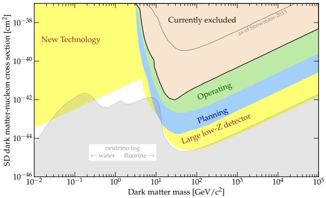
Candidates for dark matter

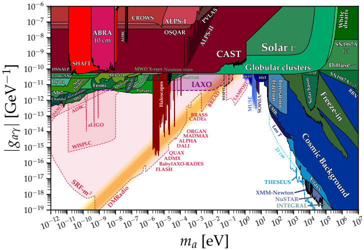


Candidates for dark matter



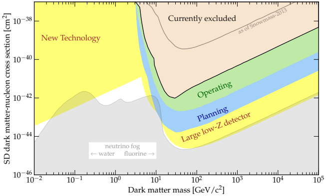
Demystifying the nature of dark matter

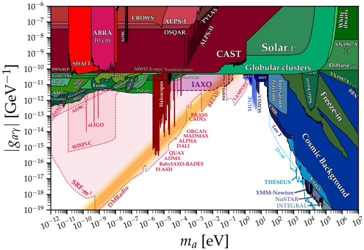




- Option 1: Hope that dark matter has some interaction to the Standard Model.

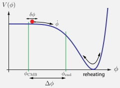
Demystifying the nature of dark matter



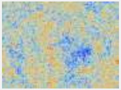


- Option 1: Hope that dark matter has some interaction to the Standard Model.
- Option 2: Do not assume any interaction other than gravitational, and figure out the nature of dark matter indirectly. For example, via structure formation.

Brief history of structure formation

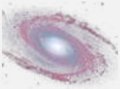


1. Inflationary perturbations and reheating



2. Temperature fluctuations in the thermal plasma

3. Dark matter density fluctuations



6. Formation of stars and galaxies

5. Collapse of baryons in DM halos

4. Collapse of dark matter fluctuations

From inflation to power spectrum

The distribution of all components in the universe are approximately homogeneous and isotropic. The deviations from the homogeneity are represented by the density δ and velocity θ perturbations:

$$\begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\theta} \end{bmatrix} = [\delta_\gamma, \delta_c, \delta_b, \delta_\nu, \ldots, \theta_\gamma, \theta_c, \theta_b, \theta_\nu, \ldots]^T$$

The time evolution of these perturbations can be calculated using cosmological perturbation theory: CLASS CAMB

$$\frac{d}{dt}\begin{bmatrix} \boldsymbol{\delta}(t) \\ \boldsymbol{\theta}(t) \end{bmatrix} = \begin{pmatrix} \cdot & \cdots & \cdot \\ \vdots & \ddots & \vdots \\ \cdot & \cdots & \cdot \end{pmatrix} \begin{bmatrix} \boldsymbol{\delta}(t) \\ \boldsymbol{\theta}(t) \end{bmatrix}, \qquad \text{Inflation model} \Rightarrow \begin{bmatrix} \boldsymbol{\delta}(t=0) \\ \boldsymbol{\theta}(t=0) \end{bmatrix}$$

Matter power spectrum

The correlation between the matter energy density perturbations is

$$\langle \delta_m(\mathbf{k},t)\delta_m(\mathbf{k}',t)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')\underbrace{P_m(k,t)}_{\text{Power spectrum}}$$

The power spectrum can be expressed conveniently as

$$P_m(k,t) = \frac{2\pi^2}{k^3}\underbrace{\mathcal{P}_{\rm inf}(k)}_{\text{initial conditions}}\;\underbrace{T^2(k,t)}_{\text{subsequent evolution}}$$

Initial conditions are imprinted by inflation:

$$\mathcal{P}_{\rm inf}(k) = A_s\left(\frac{k}{k_\star}\right)^{n_s-1}, \quad A_s \simeq 2.1\times 10^{-9}, \quad k_\star = 0.05\,\mathrm{Mpc}^{-1}.$$

Constraints on the matter power spectrum

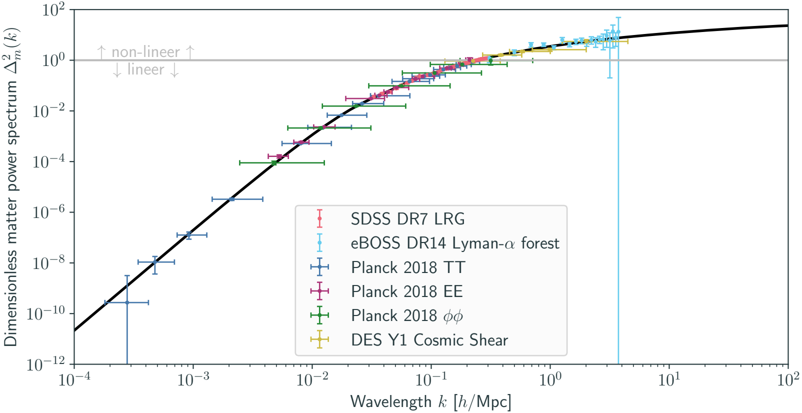


Figure 3: For the experimental data, see this GitHub repo.

Any questions?